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Modelling the Time Correlation in Hourly Observations of Direct Radiation in Clear Skies

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SUMMARY

In the paper we discuss the formulation and estimation of a stochastic model for the direct radiation in clear skies. The only external variable in the model is the solar elevation. An important feature of the method is that the strong hourly correlation that is not explicable by the solar elevation is taken into account by the model.

The application of the method is illustrated on a series of hourly values of direct radiation in periods of clear skies during eight years at a location near Copenhagen, Denmark. The periods of clear skies have been selected by means of hourly observations of cloud cover. The direct radiation has not been recorded, but calculated from observations of global and diffuse radiation. Maximum likelihood estimates of the parameters in the model are found by numerical methods, and an indication of the estimation error under the model is given. Moreover, the consequences of neglecting the correlation structure at the estimation is demonstrated.

Finally, two examples show the importance of modelling the correlation structure in applications of the model for simulation and forecasting purposes.

INTRODUCTION

For many applications in urban planning and building design, information about the variation of solar radiation is required. This information may consist of real radiation data or of a model. Real data for such applications are commonly selected so that they show maximal similarity with long-term average values for the corresponding seasonal period. One large group of climate information constructed this way is the so-called Reference Years (e.g., see ref. 1). The similarity criteria used in selecting the samples of real data usually imply that only the mean value variation is described. Hence, the application of such data may lead to a wrong conclusion to problems, where the solution in fact depends on the whole range of observed values as well as the dynamic variations of the radiation.

The alternative approach is to use a mathematical model for the variation in solar radiation. In a given model, the variation in solar radiation is determined by the variation in the external variables. Various models, on a very broad scale of complexity, have been proposed in the literature. The more complex models that have been proposed for a cloudless and clean atmosphere (e.g., see refs. 2 and 3), and which explicitly take into account the absorption by water vapor, the aerosol scattering and absorption, etc., are indispensable for understanding the effect of changes in atmospheric conditions and for very detailed calculations. But, for most practical applications, simplified models are required with a limited number of external variables, like, for example, cloud cover and solar elevation.

In the case of cloudy conditions, it is almost impossible to calculate the solar radiation at ground level based on detailed mathematical models of the physical relations. According to Paltridge and Platt [3] the accuracy of cloud specification is so poor that there is often no point in accounting for the input variations which arise from changes in water vapor, dust and ozone. The

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requirement of simplified models is thus beyond dispute.

When simple models are used, it is just as important as in the case of a reference year that the model output describes the whole range of observed values, as well as the dynamical characteristics. The dynamical behaviour may be characterized by the correlation in time. Gringorten [4] has found that the hour-to-hour correlation is around 0.95 for the major part of the surface weather observations taken at Minneapolis, Minnesota. By considering a simple deterministic model which explicitly only contains, for example, the solar elevation as an external variable, obviously only a part of the variation in radiation is described, since the well-known dependence on water vapor, aerosols, etc., is neglected. Moreover, since the inertia of variations of these external variables further contributes to hour-to-hour correlation of the solar radiation, such simplified deterministic models describe only a part of the correlation of the solar radiation.

In the present paper a simple stochastic model for the variation of direct radiation in clear skies is proposed. The model uses only one external variable — the solar elevation. But, by using the stochastic approach, the model is able to give an indirect description of the effect due to variations of other variables, like the variations of the water vapor in the atmosphere. The dependence of the solar elevation is described in a deterministic part of the model. The basic nature of the relation between direct radiation and the solar elevation is similar to the relationship in many other simplified models for technical applications [5 - 7].

The objective of the paper is to describe the formulation and estimation of a stochastic model for the direct radiation in clear skies, which describes the whole range of observed values and the total correlation in time. The consequences of neglecting that part of the variation and correlation, which is not describable by the solar elevation, is demonstrated. Finally, the ability of the model for both simulation and forecasting is illustrated, and it is shown that, for these applications, it is of decisive importance to use models which take into account the range of observed values as well as the correlation in time.

THE DATA

The radiation measurements are from the Climate and Water Balance Station on the experimental farm, Højbakkegård, situated about 20 km west of Copenhagen. The climate station is freely exposed and surrounded by ordinary agricultural fields, and the experimental site is covered by a dense and short clover-grass mixture. The global and diffuse radiation on the horizontal plane are measured by Kipp and Zonen solarimeters placed one metre above the ground. The solarimeters are identical except that the solarimeter for diffuse radiation measurements is provided with a ring to eliminate the direct part of the global radiation. The solarimeters measure the radiation between 0.3 μm and 2.5 μm. Once a year the solarimeters are compared with the Ångstrøm compensation pyrheliometer, or with a newly calibrated solarimeter.

Cloud-cover observations are taken at the synoptic observation station at Vaerløse Airport, about 15 km north of the radiation measuring site. The cloud-cover observations follow the WMO code [8]. In the present investigation the hourly observations of total cloud cover are used. Thus, the cloud cover is measured as an integer between 0 and 9, where 0 corresponds to a completely clear sky, 8 to a completely overcast sky, and the observation 9 indicates that the cloud cover is unobservable which usually is the case in foggy weather or heavy snowfall. The observations are based on a subjective evaluation, and as such they may be encumbered with judgemental errors.

The output from the radiometers are sampled every 10 minutes and from the sample values hourly averaged radiation values are constructed. Thus, the observation at time t of a radiation component represents the average radiation in the period $(t-1, t)$. The observation of the cloud cover c'_t, however, is the instantaneous cloud cover at time t. Hence, in order to obtain a representative value for cloud cover in the period $(t-1, t)$, we introduce

$$c_t = 1/2(c'_t + c'_{t-1}) \tag{1}$$

It is noticed that eqn. (1) implies that c_t takes values belonging to the set $\{0, 0.5, 1, 1.5 \ldots 8, 8.5, 9\}$. Likewise, the solar elevation corresponding to the observation of radiation at time t is calculated by using the time $t - 0.5$ h.

In this paper only models in clear skies are considered. Thus, an operative definition of clear skies is needed. It is natural to consider the observation of a radiative component given at time t referring to clear sky radiation if the cloud cover observation c'_t is 0 both at time t and at time $(t-1)$, i.e., we use the definition that $c_t = 0$ corresponds to a clear sky.

The observations used are the hourly values from February 1, 1966, to December 31, 1973, such that the total number of hourly observations is 69 384. Only 1183 observations of the total set correspond to daytime clear sky observations.

FORMULATION OF A STOCHASTIC MODEL

The fundamental relationship between direct (I_N), diffuse (I_D) and global (I_G) radiation is [3]

$$I_G = I_N \sin(h) + I_D \quad (2)$$

where h is the solar elevation. If models are given for two of the components, the model for the third component follows from eqn. (2). In the present investigation the relationship in eqn. (2) is used to calculate hourly values of direct radiation based on the hourly measurements of global and diffuse radiation.

The 1183 observations of direct radiation in clear skies — calculated by eqn. (2) from the observations of global and diffuse radiation — are shown in Fig. 1 as a function of the solar elevation. At the first glimpse the distribution of the observations seems to be rather scattered. This reflects a rather large variation in direct radiation corresponding to a given solar elevation. But, for the observations corresponding to a specific day containing several clear sky observations, a systematic pattern is recognized. In most cases none, or only a few observations corresponding to clear skies, occur within a specific day, and these observations are randomly distributed in time over the day. However, days with totally clear skies during the entire day exist. In Fig. 1, observations corresponding to each of two days of totally clear skies have been connected. The variation on the two days of totally clear skies, suggests that a model of the dependence on the solar radiation, like the model described by the curve in the middle of the observations in Fig. 1, can describe a part of the variation and correlation in time. But, the deviation from that curve may be considerable, and apparently the hour-to-hour correlation of the deviation is rather high. This hourly correlation has to be considered in order to obtain a realistic and applicable model.

The main part of the variations observed in Fig. 1 is due to the variation in time of the absorption and scattering properties of

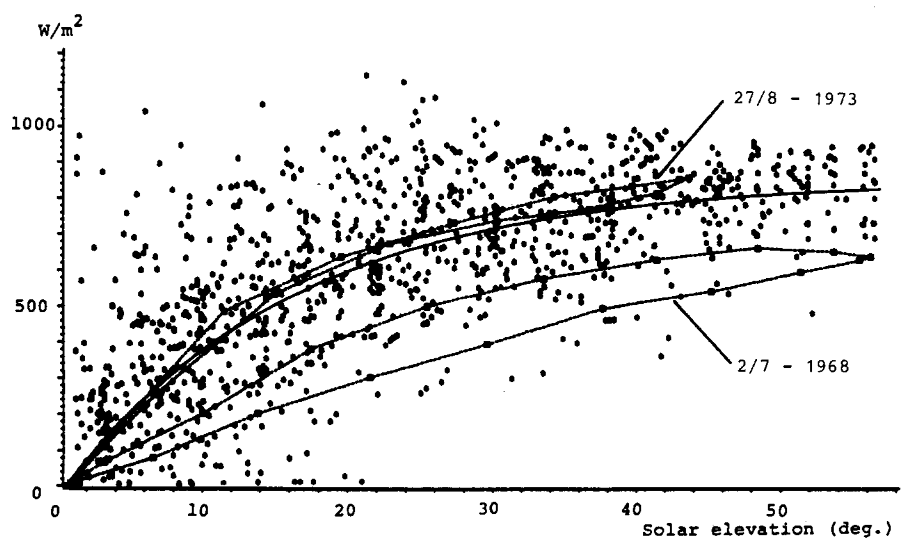

Fig. 1. Hourly observations of direct radiation in clear skies. Observations on August 27, 1973, are connected, and similarly for the observations on July 2, 1968. The curve in the middle of the observations corresponds to the final model in clear skies.

the atmosphere — only a very limited part is due to the annual variation in the distance between the sun and the earth. Therefore, well-known formulas for absorption and scattering of monochromatic radiation in a medium may be used as a starting point for constructing models for the direct radiation. This approach is briefly discussed in the following Section where the deterministic part of the model is formulated. A stochastic model, which includes a model for the deviation from the deterministic part, is constructed afterwards.

The deterministic part

In the absence of scattering, the depletion of the monochromatic parallel beam radiation is described by Beer's law [9]:

$$I_\lambda = I_{0\lambda} \exp\left(-\int \kappa_{a\lambda} \rho \, ds\right) \tag{3}$$

where $I_{0\lambda}$ is the initial radiation, ρ is the density and $\kappa_{a\lambda}$ is the spectral dependent mass absorption coefficient. The integration in eqn. (3) is along the path of radiation. Both $\kappa_{a\lambda}$ and ρ are often functions of the position along the path. The optical depth $\tau_{a\lambda}$ for the absorption at wavelength λ is defined as

$$\tau_{a\lambda} = \int \kappa_{a\lambda} \rho \, ds \tag{4}$$

It is possible to define an expression analogous to Beer's law for the scatter [3]. Hence in a pure scattering medium the depletion is described by

$$I_\lambda = I_{0\lambda} \exp\left(-\int \beta_{sc} \, ds\right) \tag{5}$$

where β_{sc} is the volume scatter coefficient. If both absorption and scatter are present in the medium we find

$$I_\lambda = I_{0\lambda} \exp[-(\tau_{a\lambda} + \tau_{sc})] \tag{6}$$

where

$$\tau_{sc} = \int \beta_{sc} \, ds \tag{7}$$

is the optical depth for scatter, and the path of integration is as in eqn. (3).

The direct radiation is the parallel beam radiation from the sun at ground level. When the direct radiation is considered, some vertical optical depth is usually defined, and the monochromatic direct radiation is then written

$$I_{N,\lambda}(h) = I_{0\lambda} \exp(-\tau_{e\lambda} m_r(h)) \tag{8}$$

where $I_{0\lambda}$ is the radiation with wavelength λ outside the atmosphere of the earth, and $\tau_{e\lambda}$ is the sum of vertical optical depth from ground level, which describes the extinctions due to Rayleigh scatter, ozone absorption, water vapor absorption and extinction (both scatter and absorption) by dust and larger particles.

The term $m_r(h)$ is the relative optical air mass, which is a function of the solar elevation h, and defined as

$$m_r(h) = \int_0^\infty \rho \, ds \Big/ \int_0^\infty \rho \, dz \tag{9}$$

where the integration in the numerator is along the path of the direct radiation and the integration in the denominator is vertical. The z-axis points out from the surface with the origin at the ground. If the atmosphere is considered as a nonrefracting plane-parallel medium, then $dz = \sin(h)\, ds$, and the relative optical air mass thus becomes

$$m_r(h) = 1/\sin(h) \tag{10}$$

Several other, more accurate approximations of the relative optical air mass, which also consider the curvature and refraction of the real atmosphere, are proposed in ref. 3.

By carrying out an integration of eqn. (8) over the whole solar spectrum, the direct radiation at the ground is obtained. A calculation of the amount of direct radiation at the ground based on the above equations is, however, rather difficult, since it requires a full knowledge of the absorption and scattering coefficients as a function of wavelength for all atmospheric constituents, and a knowledge of how these coefficients vary with temperature and pressure. The result is that empirical relationships are used in practical situations. An example of this approach has been given by Taesler and Andersson [10], who describe a method for computing the direct, diffuse and global radiation using routine meteorological observations. Paltridge and Platt [3] give a discussion of the difficulties, and they propose and discuss applicable empirical models for the absorption and scatter mechanisms in the atmosphere. These models are, however, not directly applicable in the present investigation, since the proposed models are only valid in some standard or mean value situation, and since the models, in general, are only valid for a single subprocess, e.g., water vapour absorption, and not in a global model for the whole atmosphere. A further drawback is that the local variations in the atmospheric constituents can be quite large. As an example, the difference between the incident radiation at ground level can be about 20% lower in urban areas than in corresponding rural areas [11].

Due to the difficulties with the multi-parameterized approach described above, practical approaches often attempt to describe the overall transmission properties of the atmosphere in terms of one, or at most two coefficients in simplified models for the relationship between direct radiation and the solar elevation [5]. Hallgreen [12] and Kimura [7] apply a model of the form

$$I_N(h) = I_0' k_1^{1/\sin(h)} \tag{11}$$

where I_0' and k_1 are constants. Lund [6], Russo [13] and Page [5] suggest

$$I_N(h) = I_0' \exp(-k_2 m_r'(h)) \tag{12}$$

where $m_r'(h)$ expresses an approximation to the relative optical air mass. As an example, the approximation suggested by Lund is given by

$$m_r'(h) = 1.01/(\sin(h) + 0.01) \tag{13}$$

To the extent that the atmosphere can be approximated as a non-refracting plane-parallel medium, the relative optical air mass equals $1/\sin(h)$. By this approximation the model given in eqn. (11) equals the model in eqn. (12), with $k_1 = \exp(-k_2)$.

Paltridge and Platt [3] suggest the following model

$$I_N(h) = a_N[1 - \exp(-b_N h)] \tag{14}$$

Furthermore, they suggest $a_N = 1000$ W/m^2 and $b_N = 0.06$ as generally applicable, with perhaps some correction for regions of excessive dust pollution and for extremes of total precipitable water.

Equation (8), which is based on Beer's law, is not valid when applied to overall bands since $\tau_{e\lambda}$ strongly depends on the wavelength. Therefore, the analogy with Beer's law in eqn. (12) does not necessarily imply that eqn. (12) is more attractive than eqn. (14). However, eqn. (12) may be used, if k_2 is considered to be a function of $m_r(h)$ or h [3, 14]. In the following the model (eqn. (14)) suggested by Paltridge and Platt will be used.

The stochastic part

The deterministic dependence on the solar elevation is not able to describe either the total variation or the total correlation in time of the observations in Fig. 1. Hence, the model is expanded to the following stochastic model

$$I_N(h(t)) = I_{0,N}(h(t)) + \epsilon_N(t) \tag{15}$$

$$I_{0,N}(h(t)) = a_N[1 - \exp(-b_N h(t))] \tag{16}$$

The deviation from the deterministic part $I_{0,N}$ is described by the random variable $\epsilon_N(t)$. The index 0 is introduced in eqn. (16) to indicate that $I_{0,N}$ only describes a part of the variations of I_N.

In order to get some idea of the variance structure for $\epsilon_N(t)$ it is recalled that the direct radiation is calculated from the observations of global and diffuse radiation as

$$I_N(h) = \frac{1}{\sin(h)}(I_G(h) - I_D(h)) \tag{17}$$

Based on analysis of the variations of the measurements of global radiation and diffuse radiation, it seems reasonable to assume that the variance of $I_G(h)$ is independent of h and considerably larger than the variance of $I_D(h)$. Hence the following heteroscedastic *variance structure* is assumed for clear sky direct radiation

$$V[I_N(h(t))] = V[\epsilon_N(t)] = \frac{\sigma_N^2}{\sin^2(h(t))} \tag{18}$$

That is, the variance of I_N increases as the solar elevation h decreases. This tendency is recognized in Fig. 1, where some of the values for small solar elevations in fact are rather doubtful.

The trajectories of $I_N(h)$ on the totally cloudless days depicted in Fig. 1 have a general character as the observations, for example, stand in the same part of the total ensemble during the individual days, and even closely resemble the average value trajectory. This indicates that the noise sequence $\{\epsilon_N(t)\}$ is strongly correlated. Since the variation of $I_N(h)$ within a single day is rather smooth and the daily observations mainly lie either below or above a given mean curve for all the observations, it seems reasonable, as a first approximation, to assume that the correlation is an exponential decaying function of the time distance between two observations (a first-order Markov assumption). Thus, the following *correlation structure* is suggested:

$$\mathrm{Cor}[\epsilon_N(t_i), \epsilon_N(t_j)] = \rho^{|t_i - t_j|} \tag{19}$$

where ρ is the hour-to-hour correlation. If a model in the class of ARIMA models [15] is wanted, this correlation structure implies that the random variable

$$e(t) = \epsilon_N(t)/\sin(h(t)) \tag{20}$$

follows the AR(1)-process

$$e(t) = \alpha e(t-1) + \xi(t) \tag{21}$$

where $\{\xi(t)\}$ is white noise, i.e., $\xi(t)$, $\xi(t-1)$ ... are uncorrelated.

If we furthermore assume that $\epsilon_N(t)$ is Gaussian distributed, the total *model for direct radiation in clear skies* can be summarized as

$$I_N(h(t)) = I_{0,N}(h(t)) + \epsilon_N(t) \tag{22}$$

$$I_{0,N}(h(t)) = a_N[1 - \exp(-b_N h(t))] \tag{23}$$

$$\mathrm{E}_N = (\epsilon_N(1) \ldots \epsilon_N(n))^T \in N_n(0, \sigma_N^2 \boldsymbol{\Sigma}) \tag{24}$$

$N_n(0, \sigma_N^2\boldsymbol{\Sigma})$ denotes the n-dimensional normal distribution with mean zero and covariance matrix $\sigma_N^2\boldsymbol{\Sigma}$, with the $n \times n$ dimensional matrix $\boldsymbol{\Sigma}$ given by

$$\{\Sigma_{ij}\} = \frac{\rho^{|t_i - t_j|}}{\sin(h(t_i))\sin(h(t_j))} \tag{25}$$

where n is the number of observations.

In the following the consequences of neglecting the variance and the correlation structure will be illustrated. If only the variance structure is considered in the model, the matrix $\boldsymbol{\Sigma}$ is given by

$$\{\Sigma_{ij}\} = \frac{1}{\sin(h(t_i))\sin(h(t_j))} \tag{26}$$

i.e., deviations from $I_{0,N}$ are assumed to be independent of each other, but the variance increases with decreasing solar elevation.

If only the correlation structure is considered in the model, the matrix becomes

$$\{\Sigma_{ij}\} = \rho^{|t_i - t_j|} \tag{27}$$

i.e., deviations from $I_{0,N}$ are assumed to vary independently of the solar elevation, but with a strong correlation between successive hourly values.

If neither the variance nor the correlation structure is considered, we have

$$\boldsymbol{\Sigma} = \mathbf{I} \tag{28}$$

where $\mathbf{I}$ is the unit matrix.

THE ESTIMATION METHOD

The maximum likelihood (ML) estimates of $\theta = (a_N, b_N)^T$ and σ_N^2 are found by

$$\min_\theta (\underline{I}_N - \underline{I}_{0,N}(\theta))^T \boldsymbol{\Sigma}^{-1} (\underline{I}_N - \underline{I}_{0,N}(\theta)) = \min_\theta \|\underline{I}_N - \underline{I}_{0,N}(\theta)\|^2_{\Sigma} \tag{29}$$

$$\hat{\sigma}_N^2 = \frac{1}{n} \|\underline{I}_N - \underline{I}_{0,N}(\theta)\|^2_{\Sigma,\, \theta = \hat{\theta}} \tag{30}$$

where

$$\underline{I}_N = (I_N(h_1) \ldots I_N(h_n))^T$$

$$\underline{I}_{0,N} = (I_{0,N}(h_1) \ldots I_{0,N}(h_n))^T$$

There is no explicit solution to the minimization problem. Therefore an iterative procedure, a so-called relaxation method [16], has been used.

The minimization in eqn. (29) requires an inversion of the $\boldsymbol{\Sigma}$ matrix. Since the dimension of $\boldsymbol{\Sigma}$ is 1183, a direct inversion would imply considerable efforts. In order to avoid a direct inversion the exponential decay of the autocorrelation function in eqn. (19) is cut off, such that

$$\mathrm{Cor}[\epsilon_N(t_i), \epsilon_N(t_j)] = 0 \text{ for } |t_i - t_j| \geq 100 \tag{31}$$

Since the series of observations of direct radiation in clear skies contains many successive observations in the series with a time gap that is wider than 100 hours, the convention of eqn. (31) implies that the inverse $\boldsymbol{\Sigma}$ matrix can be calculated as

$$\boldsymbol{\Sigma}^{-1} = \begin{bmatrix} \boldsymbol{\Sigma}_{11}^{-1} & 0 & 0 & 0 & \ldots & 0 \\ 0 & \boldsymbol{\Sigma}_{22}^{-1} & 0 & 0 & \ldots & 0 \\ 0 & 0 & \boldsymbol{\Sigma}_{33}^{-1} & 0 & \ldots & 0 \\ & & \cdot & & & \cdot \\ 0 & 0 & 0 & \ldots & 0 & \boldsymbol{\Sigma}_{kk}^{-1} \end{bmatrix} \tag{32}$$

Since the maximum dimension of the submatrices turned out to be 79, the cut off, eqn. (31), of the exponential decaying implies a drastic reduction of the necessary computer resources as compared to the direct inversion of $\boldsymbol{\Sigma}$.

The variance of the estimates is found as the following approximation to the inverse Fisher information matrix:

$$\hat{V}(\hat{\theta}) = 2\hat{\sigma}_N^2 \left[\frac{\partial^2}{\partial \theta^2} \, \|\underline{I}_N - \underline{I}_{0,N}(\theta)\|_{\Sigma}^2 \right]^{-1}_{|\theta = \hat{\theta}} \quad (33)$$

It is, however, important to notice that the variance derived from the Fisher information matrix does not necessarily make sense if the model is wrong.

THE RESULTS

The ML estimates of $\theta = (a_N, b_N)^T$ and σ_N^2 are shown in Table 1. With the intention to illustrate the consequence of neglecting either the variance structure or the correlation structure, the estimates corresponding to the four cases with $\boldsymbol{\Sigma}$ given by eqns. (25) - (28), respectively, are shown.

For the final model, i.e., the model where $\boldsymbol{\Sigma}$ is given by eqn. (25), which contains a consideration to both the variance and correlation structure, the estimated hour-to-hour correlation becomes

$\hat{\rho} = 0.859$

This indicates that the hourly correlation in the deviation from the deterministic dependence on the solar elevation is high — as expected. An investigation of the autocorrelation functions for the normalized residuals has shown that the first-order Markov assumption, eqn. (19), for the correlation structure is reasonable [17].

A comparison between the model without consideration to any structure for $\boldsymbol{\Sigma}$ ($\boldsymbol{\Sigma} = \mathbf{I}$) and the final model is shown in Fig. 2. Paltridge and Platt have suggested the values $a_N = 1000$ W/m^2 and $b_N = 0.06$ as generally applicable. A model with these values is also depicted in Fig. 2.

Figure 2 shows a large difference between the final model and the model suggested by Paltridge and Platt — the difference is about 150 W/m^2 at high solar elevations. There are several possible explanations. First of all, Paltridge and Platt noticed that the coefficients had to be adjusted in regions of high dust pollution and for extremes of precipitable water in the atmosphere.

Results given by Petersen (ref. 14, p. 27), and observations at Vaerløse Airport about 15 km north of Højbakkegård, suggest that the direct radiation at the time of high solar elevation, say 50°, is about 30 W/m^2 lower at Vaerløse than in rural areas. Since the distance from Højbakkegård to the major pollutant, Copenhagen, is nearly the same as for Vaerløse, the effect of air pollution is probably around 30 W/m^2 for both sites.

Secondly, a systematical error exists in the measurements of radiation which depends on the air temperature. According to Hansen *et al.* [18] the error is about −0.15% per °C. If a mean temperature of 20 °C is assumed at the time of large solar elevations, this error will account for approximately another 30 W/m^2.

TABLE 1

The maximum likelihood estimates of the parameters in the model for direct radiation in clear skies with different covariance structures. The standard deviation of the estimates are shown in brackets

Consideration to	$\boldsymbol{\Sigma}$ given by	$\hat{a}_N$ (W/m^2)	$\hat{b}_N$	$\hat{\sigma}_N^2$ ((W/m^2)^2)
— ($\boldsymbol{\Sigma} = \mathbf{I}$)	Eqn. (28)	798.6 (13.1)	0.0798 (0.0043)	34631.8
Variance structure	Eqn. (26)	822.6 (5.2)	0.0706 (0.0012)	3947.6
Correlation structure	Eqn. (27)	827.8 (7.3)	0.0551 (0.0022)	25180.1
Variance and correlation structure	Eqn. (25)	842.3 (7.4)	0.0614 (0.0013)	2302.1

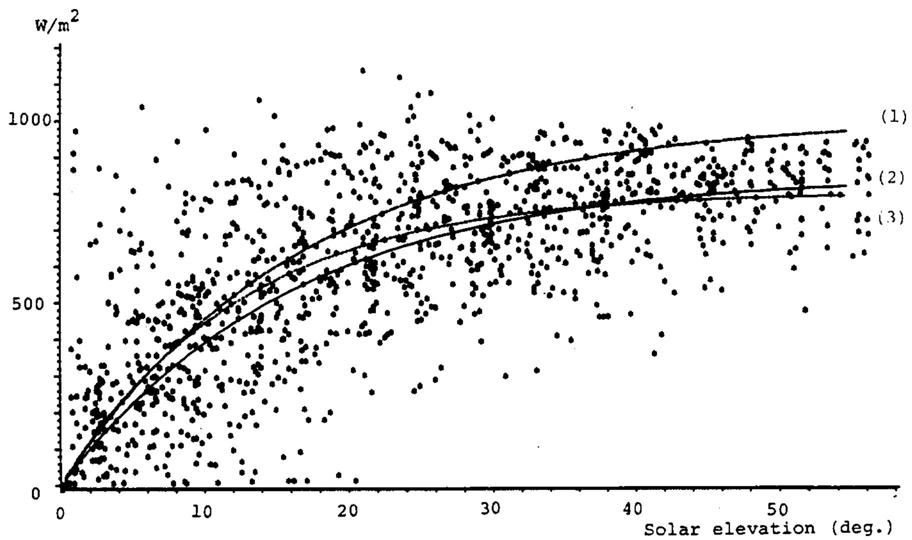

Fig. 2. Direct radiation in clear skies. Data taken from 1966 until 1973 in all months. Final model (2), model without consideration of variance and correlation structures (3), and the model based on the coefficients suggested by Paltridge and Platt in 1976 (1).

Finally, it should be noted that the coefficients proposed by Paltridge and Platt have been estimated and based on observations from Aspendale in Colorado, where the annual mean atmospheric water content is 1.5 cm [3] and less than the mean atmospheric water content for Højbakkegård, which roughly estimated is 3 - 4 cm.

The largest difference between results obtained by the final model and the model without any structure ($\mathbf{\Sigma} = \mathrm{I}$) is about 50 W/m^2 and occurs near a solar elevation of 12°. This difference occurs mainly because the model without any structure gives the large and very doubtful values for small solar elevations the same weight as the more credible observations for large solar elevations, in opposition to the final model where the influence of the doubtful observations is scaled down. Compared to the total variation of the observations, the difference between the models is rather small.

THE EFFECT OF TIME-CORRELATION MODELLING IN MODEL PERFORMANCE

Many applications of climate models in urban planning and building design is in simulation. Another kind of application is in forecasting of climate variables, e.g., for an improved control of the energy supply to buildings. In both cases a suitable model of the correlation and variance structure is crucial. This is illustrated in the following two simple examples.

The first example illustrates the importance of a proper correlation structure when the model is used for simulation. Both curves (1) and (2) in Fig. 3 illustrate simulated direct radiation in clear skies. The dots are the 1183 observations and the curve in the middle of the observations corresponds to the deterministic part of the final model, i.e., $I_{0,\mathrm{N}}$. Curve (1) shows a simulation based on $\rho = 0$, whereas the second curve shows a simulation based on the estimated correlation, $\rho = 0.859$. The variance structure from eqn. (18) is used in both cases, hence curve (1) corresponds to $\mathbf{\Sigma}$ given by eqn. (26) and curve (2) corresponds to $\mathbf{\Sigma}$ given by eqn. (25). The simulation starts at sunrise and stops at noon.

The simulation based on a model without the proper correlation structure is seen to be unrealistically fluctuating around $I_{0,\mathrm{N}}$ which is the deterministic part of the model. This is in conflict with the observed variation on the two specific days in Fig. 1. Conversely, the result of the simulation based on a model with a reasonable correlation structure is more able to reproduce the observed inertia in sequences of successive observations.

The second example considers another case of application, namely forecasting. Suppose the time is July 2nd at 10:00. Up to this time the observed direct radiation

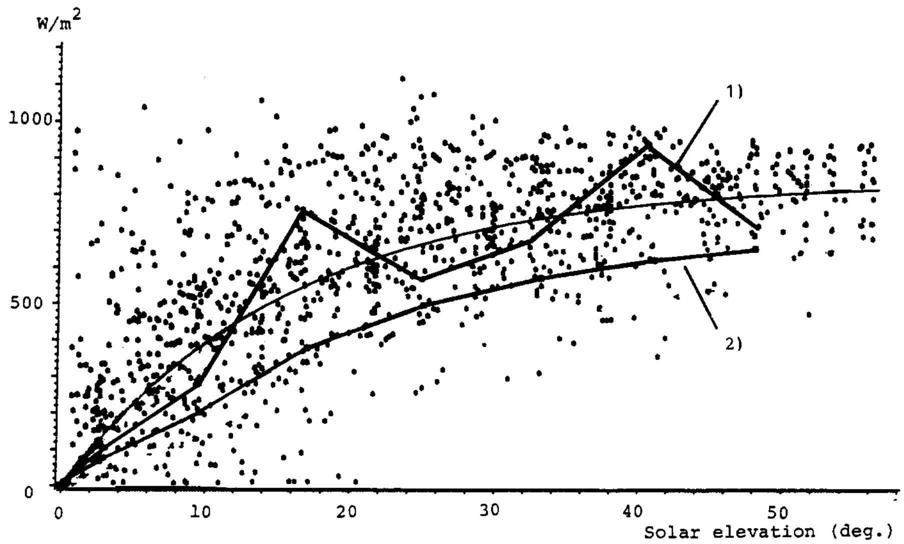

Fig. 3. Simulation of direct radiation in clear skies. Curve 1) hour-to-hour correlation equal to 0; curve 2) hour-to-hour correlation equal to 0.859.

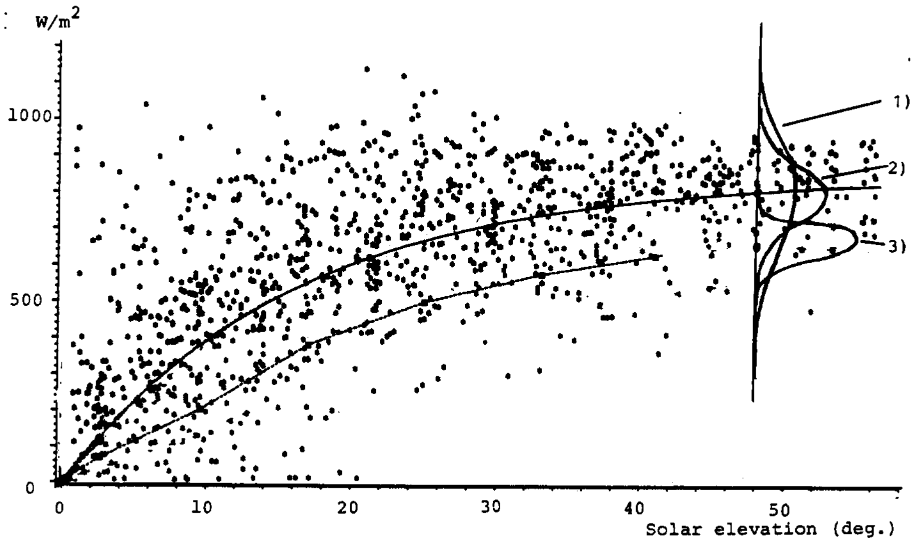

Fig. 4. Forecasting of direct radiation in clear skies. Curve 1) no covariance structure; curve 2) only variance structure; curve 3) both variance and correlation structure.

has been as indicated by the dashed curve in Fig. 4, with the solid curve representing the mean value variation of the direct radiation in clear skies, as described by $I_{0,N}$. Suppose now that a forecast of the direct radiation at 11:00 is wanted, and that a calculation has shown that the solar elevation will be about 48° at that time. In the case of a model without any covariance structure ($\mathbf{\Sigma} = \mathbf{I}$), the forecast is the point on the solid curve $I_{0,N}$ corresponding to $h = 48°$. This prediction is 798.1 W/m^2. The calculated standard deviation of the prediction under this model is 186.1 W/m^2 (= $\sqrt{34631.8}$ — see Table 1). If the prediction error is assumed to be Gaussian distributed, the derived distribution of the direct radiation at 11:00 becomes as illustrated by distribution curve 1) in Fig. 4.

If a model, which describes the variance structure properly ($\mathbf{\Sigma}$ given by eqn. (26)) is used, the forecast is again 798.1 W/m^2, but the calculated standard deviation of the prediction under this model is 84.5 W/m^2 — see distribution curve 2) in Fig. 4. If both the correlation and variance structure are considered ($\mathbf{\Sigma}$ given by eqn. (25)), the forecast will utilize the observation at 10:00, and the derived distribution of the radiation at 11:00 becomes as indicated by distribution curve 3). The calculated standard deviation of the prediction is reduced to 64.5 W/m^2. Due to the high hourly correlation the forecast is now about 690 W/m^2. This is clearly below the solid curve $I_{0,N}$ in accordance with the tendency up to 10:00. The forecasts without consideration to the correlation structure are obviously about 108 W/m^2 too large.

CONCLUSION

Due to the fact that many phenomena show correlation in time, a method is needed for formulation and estimation of models, which takes into account this correlation structure. In the present paper a method is illustrated, which systematically takes into account both the variance and correlation structure, and it is demonstrated that for practical applications a reasonable description of the correlation structure is crucial. In the actual case the formulation of a stochastic model for the variation of direct radiation in clear skies is considered.

The analysis of almost eight years of hourly observations of direct radiation in clear skies has shown a strong correlation in time, and a stochastic model for the observed variation is formulated. The only external variable is the solar elevation, which enters the model in a deterministic part describing a very simple relationship between the direct radiation and the solar elevation. The formulation and estimation of the model is discussed. By the stochastic approach it is possible to describe the whole range of observed values and the strong correlation in time. Examples have shown that a consideration to a proper structure of the variance and correlation becomes utterly important in applications of the model.

For building and energy applications a total model for radiation must include models of diffuse radiation, and a description of the dependence on cloud cover. Stochastic models for the variation of direct and diffuse radiation are separated by Madsen [17] into clear sky models for each of the radiative components, and models which account for the dependence of cloud cover. The dependence on other factors, like absorption of water vapor, is described indirectly by a description of the correlation structures. A model for global radiation is directly based on the models for direct and diffuse radiation, and it is shown that the descriptive ability of the model is quite as good as much more complex models, which explicitly take into account the variation of dust, water vapour, air pressure, etc. By using a simulation model for cloud cover [19], a total stochastic model for the variation of direct, diffuse and global radiation is then obtained.

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