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# Application of predictive control in district heating systems

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*In district heating systems, and in particular if the heat production takes place at a combined heat and power (CHP) plant, a reasonable control strategy is to keep the supply temperature from the district heating plant as low as possible. However, the control is subject to some restrictions, for example, that the total heat requirement for all consumers is supplied at any time and each individual consumer is guaranteed some minimum supply temperature at any time. A lower supply temperature implies lower heat loss from the transport and the distribution network, and lower production costs.*

*A district heating system is an example of a non-stationary system, and the model parameters have to be time varying. Hence, the classical predictive control theory has to be modified.*

*Simulation experiments are performed in order to study the performance of modified predictive controllers. The systems are, however, described by transfer function models identified from real data.*

## NOTATION

| | |
|---|---|
| $a, b, c$ | model parameters |
| $A, B, C$ | polynomials |
| $e$ | white noise |
| $E$ | expected value |
| $f(\,)$ | transfer function |
| $\mathbf{F}$ | $(NU \times NU)$-dimensional filter matrix |
| $g(\,)$ | objective function |
| $\boldsymbol{g}$ | $NU$-dimensional vector |
| $h$ | impulse response weight |
| $\mathbf{H}$ | $\{(N_2 - N_1 + 1) \times NU\}$-dimensional matrix of $h$ |
| $\mathbf{I}$ | identity matrix |
| $J$ | cost function |
| $n_A, n_B, n_C$ | order of the polynomial $A$, $B$ and $C$ |
| $N_1$ | minimum costing horizon |
| $N_2$ | maximum costing horizon |
| $NU$ | control horizon |
| $P\{\,\}$ | probability measure |
| $q^{-1}$ | back shift operator |
| $u$ | supply temperature from the CHP plant (control signal) |
| $\boldsymbol{u}$ | $NU$-dimensional vector of $u$ |
| $v$ | known part of the prediction |
| $\boldsymbol{v}$ | $(N_2 - N_1 + 1)$-dimensional vector of $v$ |
| $y$ | supply temperature in the district heating network (output signal) |
| $\boldsymbol{y}$ | $(N_2 - N_1 + 1)$-dimensional vector of $y$ |

### Greek letters

| | |
|---|---|
| $\alpha$ | service parameter (probability of too cold supply temperature) |
| $\alpha, \beta, \gamma$ | model parameters |
| $\Delta$ | change from time to time (def. $\Delta u_t = u_t - u_{t-1}$) |
| $\lambda$ | penalty parameter |
| $\boldsymbol{\Lambda}$ | $(NU \times NU)$ penalty matrix |
| $\pi$ | quantile in the standardized normal distribution |
| $\sigma^2$ | variance |
| $\omega$ | frequency |

### Superscripts and subscripts

| | |
|---|---|
| $i$ | index |
| $j$ | prediction horizon |
| $k$ | time delay |
| max | maximum value |
| min | minimum value |
| ref | reference value |
| $t$ | time |
| $t + j \mid t$ | (prediction) at time $t + j$ given the information up to time $t$ |
| T | transposed |
| $\hat{}$ | prediction |
| $\tilde{}$ | prediction error |
| $\bar{}$ | mean value |

### Abbreviations

| | |
|---|---|
| ARMAX | auto-regressive-moving-average-extraneous |
| CHP | combined heat and power |
| GPC | generalized predictive control |

## 1 INTRODUCTION

It can be shown (**1**) that considerable savings can be obtained by lowering the supply temperature from the CHP plant. These savings are due to both lower production cost at the plant, that is decreasing the supply temperature increases the ratio of power to heat output, as electricity is more valuable than heat, and so a more profitable operation is achieved [see, for example, reference (**2**)], and lower heat loss in the transport and the distribution network. On the other hand, the CHP plant has to satisfy the total heat requirement for all consumers at any time and guarantee each individual consumer some minimum supply temperature at any time.

Some other kind of constraints can be encountered, for example on the degree of time variation of the

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supply temperature from the plant. Obviously there is some kind of trade-off between the interests of the district heating company and its consumers. In this paper it is shown how optimal predictive control can be used to solve this problem.

In order to find an optimal control, a model for the district heating network is needed. If a perfect description of the entire network is available, a pure physical model can be obtained. Unfortunately, for large networks this type of model can be very difficult to establish, and if a model can be established it is most likely to be too complex for control purposes. Therefore, in this work statistical transfer functions are identified. These transfer functions describe the relations between the supply temperature from the district heating plant and the supply temperature at several representative locations in the network.

A district heating system is an example of a non-stationary system, and therefore time-varying parameters have to be introduced. The time variations of the parameters are mainly of two kinds: slow annual variations which are, for example, due to changes of the dynamic characteristics of the distribution network (induced by seasonal climate changes) and faster diurnal variations induced by varying heat consumption. The slow variations can be coped with by using adaptive estimation techniques, while the faster variations are modelled explicitly.

The time delay in district heating systems is relatively large. Since the time delay is time varying and a function of several variables, like distance, flow in the pipes, temperatures and intersection layout of the network, the identification of the delay is one of the major problems in modelling district heating systems. Methods for estimating or tracking such a time-varying time delay are developed and described in detail in references (**3**) and (**4**). Hence, this problem will not be addressed in this paper.

Because of the large time delays in the district heating systems, a predictive control is of major interest. Several types of predictive or long-range predictive controllers can be found in the literature [see, for example, references (**5**) and (**6**)], but in this paper only the generalized predictive controller (GPC), first presented in reference (**7**) for stationary systems, will be considered. Due to the non-stationarity in district heating systems, some modifications to the ordinary GPC are required.

The paper starts with a discussion of the optimum control problem at hand. This is followed by the proposed solution of the problem. Then simulation experiments are performed in order to study the performance of the controllers. Finally some conclusions are drawn.

## 2 THE PROBLEM

If $u_t$ denotes the supply temperature from the CHP plant (the control signal) and $y_t$ denotes the supply temperature in the district heating network (the output signal) (see Fig. 1) then the optimal control problem can be formulated as:

The objective is to find

$$\min_{u_t} E\{g(y_t, u_t, e_t)\} \tag{1}$$

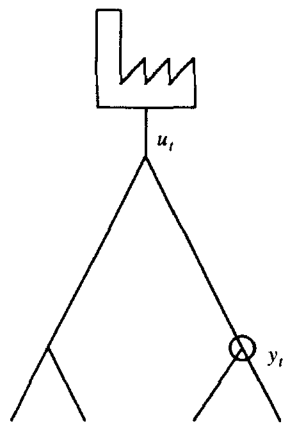


**Fig. 1** A sketch of a simple district heating network

subject to

$$y_t = f(u_t, e_t) \tag{2}$$

$$P\{y_t \leqslant y_t^{\min}\} = \alpha \tag{3}$$

$$\Delta u_t \quad \text{small} \tag{4}$$

where $E$ denotes the expected value, $g(\,)$ is an objective function, $f(\,)$ is a transfer function, $P\{\,\}$ is a probability measure, $\alpha$ is the probability of a too cold supply temperature (this is a kind of service parameter) and $\Delta u_t$ denotes the change from time to time in the supply temperature from the district heating plant ($\Delta u_t = u_t - u_{t-1}$). Notice that $y_t^{\min}$ is a function of the ambient air temperature, as illustrated in Fig. 2.

In the following the system equation (2) is assumed to be linear. The constraint in equation (3) is called the *chance constraint* in the stochastic optimization literature [see, for example, (**8**)]. If the probability $\alpha = 0$ then the constraint is called the *fat formulation.* A reference value, $y_t^{\text{ref}}$, can be obtained as the deterministic equivalent for this constraint, assuming that the error in the predictions of the supply temperature in the network and the ambient air temperature are normally distributed (and uncorrelated). This is illustrated in Fig. 2. The figure shows how the minimum supply temperature curve depends on the ambient air temperature, that is, the minimum supply temperature is constant for ambient temperatures higher than a certain value and for lower ambient air temperatures it is increasing linearly with falling ambient air temperature.

The soft constraint in condition (4) can be made more strict, that is $|\Delta u_t| \leqslant \Delta u_{\max}$. This will be discussed later on.

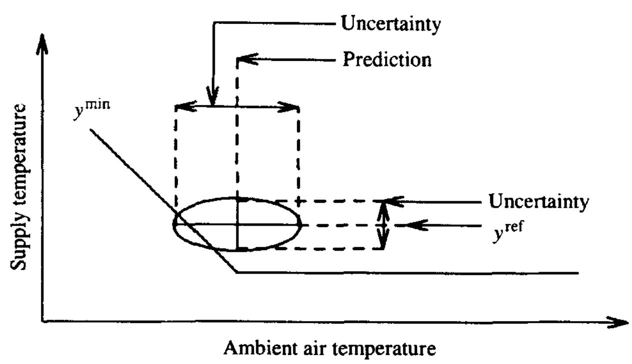


**Fig. 2** The reference value determined from the prediction uncertainties. Probability mass over the minimum curve is $100(1 - \alpha)$ per cent

In order to make use of the predictive control technique the object of the optimal control is written in the form

$$\min_{u_t} E\{(y_{t+k} - y^{\text{ref}}_{t+k|t})^2 + \lambda(\Delta u_t)^2\} \tag{5}$$

that is the controller keeps the output close to the predetermined reference output and keeps the change in the control signal small. The cost parameter $\lambda$ can be adjusted to put more or less penalty on changes in the control signal.

It can be convenient to optimize the cost function in equation (5) over several time periods. This results in the GPC cost function

$$J = E\left\{\sum_{j=N_1}^{N_2} (y_{t+j} - y^{\text{ref}}_{t+j|t})^2 + \sum_{j=1}^{NU} \lambda_{j,t}(\Delta u_{t+j-1})^2\right\} \tag{6}$$

where $N_1$ and $N_2$ are the minimum and the maximum costing horizons respectively, $\lambda_{j,t}$ is a penalty sequence, $NU$ is the control horizon and $y^{\text{ref}}_{t+j|t}$ is a reference temperature at time $t + j$.

The expectation, $E$, is conditioned on observations available at time $t$. Guidelines for determining the design parameters, $N_1$, $N_2$ and $NU$, can be found in reference (**7**).

## 3 THE SOLUTION

First the model is introduced. Then it is shown how the predictions needed in the controller are obtained. This results in a modified GPC controller.

### 3.1 Transfer function

The relation between the supply temperature from the CHP plant, $u_t$, and the supply temperature in the network, $y_t$, can be described by a time-varying ARMAX (Auto-Regressive-Moving-Average-eXtraneous) model

$$A_t(q^{-1})y_t = B_t(q^{-1})u_t + C_t(q^{-1})e_t \tag{7}$$

where $e_t$ is white noise with mean zero and variance $\sigma_e^2$. $A_t$, $B_t$ and $C_t$ are polynomials in $q^{-1}$ (the back shift operator) with time-varying coefficients:

$$\begin{aligned} A_t(q^{-1}) &= 1 + a_{1,t}q^{-1} + \cdots + a_{n_A,t}q^{-n_A} \\ B_t(q^{-1}) &= b_{1,t}q^{-1} + \cdots + b_{n_B,t}q^{-n_B} \\ C_t(q^{-1}) &= 1 + c_{1,t}q^{-1} + \cdots + c_{n_C,t}q^{-n_C} \end{aligned} \tag{8}$$

The parameter variations are periodical:

$$\begin{aligned} a_{j,t} &= \alpha_{j,0} + \alpha_{j,1}\sin\{\omega(t-j)\} + \alpha_{j,2}\cos\{\omega(t-j)\} \\ b_{j,t} &= \beta_{j,0} + \beta_{j,1}\sin\{\omega(t-j)\} + \beta_{j,2}\cos\{\omega(t-j)\} \\ c_{j,t} &= \gamma_{j,0} + \gamma_{j,1}\sin\{\omega(t-j)\} + \gamma_{j,2}\cos\{\omega(t-j)\} \end{aligned}$$

where $\omega = 2\pi/24 = \pi/12$, that is the diurnal variation for the sampling interval equal to 1 hour. If the parameters $\alpha$, $\beta$ and $\gamma$ are time varying as well, then they can be estimated adaptively using recursive methods [see, for example, reference (**9**)].

## 4 OUTPUT PREDICTION

The GPC is based on the assumption that the output predictions can be expressed as a linear combination of present and future controls. In reference (7) and many other references this is obtained by solving the Diophantine equation (sometimes recursively). However, in the time-varying case the Diophantine equation *cannot* be used due to the time variation of the model parameters. Instead the $j$-step predictor, $\hat{y}_{t+j|t}$, is found as the conditional expectation of $y_{t+j}$ conditioned on observations of the output up to time $t$ [see, for example, reference (**10**)]:

$$\hat{y}_{t+j|t} = -\sum_{i=1}^{n_A} a_{i,t+j}\hat{y}_{t+j-i|t} + \sum_{i=1}^{n_B} b_{i,t+j}u_{t+j-i} + \sum_{i=0}^{n_C} c_{i,t+j}\hat{e}_{t+j-i|t}, \qquad j \geqslant 1 \tag{9}$$

$$\hat{y}_{t+j|t} = y_{t+j}, \qquad j < 1 \tag{10}$$

where

$$\hat{e}_{t+l|t} = \begin{cases} 0, & \text{if } l \geqslant 1 \\ e_{t+l}, & \text{if } l < 1 \end{cases} \tag{11}$$

A simple example illustrates the method.

*Example*

Consider the ARX(1, 2) model ($n_A = 1$, $n_B = 2$):

$$y_t + a_{1,t}y_{t-1} = b_{1,t}u_{t-1} + b_{2,t}u_{t-2} + e_t \tag{12}$$

The one-step predictor is derived from equations (9) to (11) as

$$\begin{aligned} \hat{y}_{t+1|t} &= -a_{1,t+1}y_t + b_{1,t+1}u_t + b_{2,t+1}u_{t-1} \\ &= h_{1,t+1}u_t + v_{1,t} \end{aligned} \tag{13}$$

where $v_{1,t} = -a_{1,t+1}y_t + b_{2,t+1}u_{t-1}$ is known and $(h_{1,t+1})u_t$ is unknown before the control signal is chosen at time $t$. Note that $h_{1,t+1} = b_{1,t+1}$ is the first weight of the time-varying impulse response function.

In general, the $j$-step predictor is given as

$$\begin{aligned} \hat{y}_{t+j|t} &= -a_{1,t+j}\hat{y}_{t+j-1|t} + b_{1,t+j}u_{t+j-1} + b_{2,t+j}u_{t+j-2} \\ &= -a_{1,t+j}(h_{j-1,t+j-1}u_t + \cdots + h_{1,t+j-1}u_{t+j-2} + v_{j-1,t}) \\ &\quad + b_{1,t+j}u_{t+j-1} + b_{2,t+j}u_{t+j-2} \\ &= h_{j,t+j}u_t + \cdots + h_{1,t+j}u_{t+j-1} + v_{j,t} \\ &= \sum_{i=1}^{j} h_{i,t+j}u_{t+j-i} + v_{j,t} \end{aligned} \tag{14}$$

For the given model the $h_{i,t}$ and $v_{i,t}$ values can be computed recursively as

$$h_{i,t} = \begin{cases} b_{i,t}, & \text{if } i = 1 \\ b_{i,t} - a_{1,t}h_{i-1,t-1}, & \text{if } i = 2 \\ -a_{1,t}h_{i-1,t-1}, & \text{if } i \geqslant 3 \end{cases} \tag{15}$$

and

$$v_{i,t} = \begin{cases} -a_{1,t+i}y_t + b_{2,t+i}u_{t-1}, & \text{if } i = 1 \\ -a_{1,t+i}v_{i-1,t}, & \text{if } i \geqslant 2 \end{cases} \tag{16}$$

The coefficients $h_{i,t+j}$ ($i = 1, 2, \ldots$) are the weights of the time-varying impulse response function describing the dynamic relation between the input and the output, that is $h_{i,t}$ is the marginal change of $y_t$ changing $u_{t-i}$.

An alternative way to find the $h_{i,t+j}$ and $v_{j,t}$ values is as follows:

1. To find $v_{j,t}$, set the present and future control to zero ($u_{t+j-i} = 0$ for $i = 1, 2, \ldots, j$). Then compute $v_{j,t}$ as the conditional expectation, $\hat{y}_{t+j|t}$, using equation (9).
2. To find $h_{i,t+j}$ ($i = 1, 2, \ldots, j$), set the present and past output and past control to zero ($v_{j,t} = 0$). Feed an impulse into the system at time $t + j - i$:

$$u_{t+j-l} = \begin{cases} 1, & \text{if } l = i \\ 0, & \text{otherwise} \end{cases} \tag{17}$$

Then compute $h_{i,t+j}$ as the conditional expectation, $\hat{y}_{t+j|t}$, using equation (9).

Considering equation (14) it is seen that the $j$-step predictions, $j$ running from $N_1$ up to $N_2$, can be written as a linear matrix expression:

$$\hat{\boldsymbol{y}}_t = \mathbf{H}_t \boldsymbol{u}_t + \boldsymbol{v}_t \tag{18}$$

where

$$\hat{\boldsymbol{y}}_t = [\hat{y}_{t+N_1|t}, \ldots, \hat{y}_{t+N_2|t}]^{\mathrm{T}}$$

$$\boldsymbol{u}_t = [u_t, \ldots, u_{t+NU-1}]^{\mathrm{T}}$$

$$\boldsymbol{v}_t = [v_{N_1,t}, \ldots, v_{N_2,t}]^{\mathrm{T}}$$

$$\mathbf{H}_t = \begin{bmatrix} h_{1,t+N_1} & 0 & \cdots & 0 \\ h_{2,t+N_1+1} & h_{1,t+N_1+1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h_{NU,t+N_1+NU-1} & h_{NU-1,t+N_1+NU-1} & \cdots & h_{1,t+N_1+NU-1} \\ h_{NU+1,t+N_1+NU} & h_{NU,t+N_1+NU} & \cdots & h_{2,t+N_1+NU} + h_{1,t+N_1+NU} \\ \vdots & \vdots & & \vdots \\ h_{N_2-N_1+1,t+N_2} & h_{N_2-N_1,t+N_2} & \cdots & \sum_{i=1}^{N_2-N_1-NU+2} h_{i,t+N_2} \end{bmatrix}$$

assuming that $N_2 > NU + N_1$. The summation in the last column of the time-varying impulse response matrix, $\mathbf{H}_t$, results from the GPC constraint $\Delta u_{t+i-1} = 0, i > NU$ [see reference (**11**)].

*Remark*

In equation (18) it has been assumed that the same model is applied for all prediction horizons. Actually, this need not be the case. If different models are used, then the $j$th row of $\mathbf{H}_t$ and the $j$th element of $\boldsymbol{v}_t$ belong to a special model designed for $j$-step prediction. Making use of an individual model for each horizon is often relevant if a non-linear system is approximated by a family of linear models (for example threshold models).

### 4.1 Controller

Introducing matrix notation the cost function, equation (6), is written as

$$J = E[(\boldsymbol{y}_t - \boldsymbol{y}_t^{\mathrm{ref}})^{\mathrm{T}}(\boldsymbol{y}_t - \boldsymbol{y}_t^{\mathrm{ref}}) + \Delta\boldsymbol{u}_t^{\mathrm{T}} \boldsymbol{\Lambda}_t \Delta\boldsymbol{u}_t] \tag{19}$$

where

$$\boldsymbol{y}_t = [y_{t+N_1}, \ldots, y_{t+N_2}]^{\mathrm{T}}$$

$$\boldsymbol{y}_t^{\mathrm{ref}} = [y_{t+N_1|t}^{\mathrm{ref}}, \ldots, y_{t+N_2|t}^{\mathrm{ref}}]^{\mathrm{T}}$$

$$\Delta\boldsymbol{u}_t = [\Delta u_t, \ldots, \Delta u_{t+NU-1}]^{\mathrm{T}}$$

$$\boldsymbol{\Lambda}_t = \begin{bmatrix} \lambda_{1,t} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_{NU,t} \end{bmatrix}$$

By the projection theorem [see, for example, reference (**12**)], the output vector $\boldsymbol{y}_t$, is decomposed into two (stochastic) independent parts:

$$\boldsymbol{y}_t = \hat{\boldsymbol{y}}_t + \tilde{\boldsymbol{y}}_t \tag{20}$$

where $\hat{\boldsymbol{y}}_t$ is a vector containing the output predictions described in the previous section and $\tilde{\boldsymbol{y}}_t$ is a vector containing the prediction errors:

$$\tilde{\boldsymbol{y}}_t = [\tilde{y}_{t+N_1|t}, \ldots, \tilde{y}_{t+N_2|t}]^{\mathrm{T}} \tag{21}$$

The cost function can then be written as

$$J = [(\hat{\boldsymbol{y}}_t - \boldsymbol{y}_t^{\mathrm{ref}})^{\mathrm{T}}(\hat{\boldsymbol{y}}_t - \boldsymbol{y}_t^{\mathrm{ref}}) + \Delta\boldsymbol{u}_t^{\mathrm{T}} \boldsymbol{\Lambda}_t \Delta\boldsymbol{u}_t] + E[\tilde{\boldsymbol{y}}_t^{\mathrm{T}} \tilde{\boldsymbol{y}}_t] \tag{22}$$

Note that the last term in this expression does not depend on $\boldsymbol{u}_t$.

$\Delta\boldsymbol{u}_t$ is written as

$$\Delta\boldsymbol{u}_t = \mathbf{F}\boldsymbol{u}_t + \boldsymbol{g}_t \tag{23}$$

where

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ -1 & 1 & \cdots & 0 & 0 \\ 0 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -1 & 1 \end{bmatrix}$$

$$\boldsymbol{g}_t = [-u_{t-1}, 0, \ldots, 0]^{\mathrm{T}}$$

Using equations (18) and (23) the cost function becomes

$$J = [(\mathbf{H}_t \boldsymbol{u}_t + \boldsymbol{v}_t - \boldsymbol{y}_t^{\mathrm{ref}})^{\mathrm{T}}(\mathbf{H}_t \boldsymbol{u}_t + \boldsymbol{v}_t - \boldsymbol{y}_t^{\mathrm{ref}}) + (\mathbf{F}\boldsymbol{u}_t + \boldsymbol{g}_t)^{\mathrm{T}} \boldsymbol{\Lambda}_t (\mathbf{F}\boldsymbol{u}_t + \boldsymbol{g}_t)] + E[\tilde{\boldsymbol{y}}_t^{\mathrm{T}} \tilde{\boldsymbol{y}}_t] \tag{24}$$

The GPC control law is then obtained by minimizing the resulting cost function, subject to the constraint

$$\Delta u_{t+i-1} = 0, \qquad i > NU \tag{25}$$

This is done by setting the derivative of the cost function w.r.t. $\boldsymbol{u}_t$ to zero, that is

$$\frac{\partial J}{\partial \boldsymbol{u}_t} = 0 \tag{26}$$

In other words,

$$2\mathbf{H}_t^{\mathrm{T}}(\mathbf{H}_t \boldsymbol{u}_t + \boldsymbol{v}_t - \boldsymbol{y}_t^{\mathrm{ref}}) + 2\mathbf{F}^{\mathrm{T}} \boldsymbol{\Lambda}_t (\mathbf{F}\boldsymbol{u}_t + \boldsymbol{g}_t) = 0$$

or

$$[\mathbf{H}_t^{\mathrm{T}} \mathbf{H}_t + \mathbf{F}^{\mathrm{T}} \boldsymbol{\Lambda}_t \mathbf{F}]\boldsymbol{u}_t + [\mathbf{H}_t^{\mathrm{T}}(\boldsymbol{v}_t - \boldsymbol{y}_t^{\mathrm{ref}}) + \mathbf{F}^{\mathrm{T}} \boldsymbol{\Lambda}_t \boldsymbol{g}_t] = 0$$

Solving for $\boldsymbol{u}_t$ results in

$$\boldsymbol{u}_t = -[\mathbf{H}_t^{\mathrm{T}} \mathbf{H}_t + \mathbf{F}^{\mathrm{T}} \boldsymbol{\Lambda}_t \mathbf{F}]^{-1}[\mathbf{H}_t^{\mathrm{T}}(\boldsymbol{v}_t - \boldsymbol{y}_t^{\mathrm{ref}}) + \mathbf{F}^{\mathrm{T}} \boldsymbol{\Lambda}_t \boldsymbol{g}_t] \tag{27}$$

Only the first element of the control vector, $\boldsymbol{u}_t$, is implemented so the control law is written as

$$u_t = [1, 0, \ldots, 0]\boldsymbol{u}_t \tag{28}$$

## 5 SIMULATION RESULTS

### 5.1 Preliminaries

*5.1.1 Model and parameters*

The performance of the controller is studied by simulations. Results for various values of the control parameters are compared.

The model used is the time-varying ARMAX model shown in equations (7) and (8), where $n_A = 1$, $n_B = 4$, $n_C = 0$ and $k = 2$ [that is $b_{2,t}$ is the first non-zero parameter in $B_t(q^{-1})$]. The time variations are solely in the $b$ parameters, that is $a_1$ is constant and $b_{l,t}$ values are given by

$$\begin{aligned} b_{2,t} &= \beta_{2,0} + \beta_{2,1} \sin\{\omega(t-2)\} + \beta_{2,2} \cos\{\omega(t-2)\} \\ b_{3,t} &= \beta_{3,0} + \beta_{3,1} \sin\{\omega(t-3)\} + \beta_{3,2} \cos\{\omega(t-3)\} \\ b_{4,t} &= \beta_{4,0} + \beta_{4,1} \sin\{\omega(t-4)\} + \beta_{4,2} \cos\{\omega(t-4)\} \end{aligned} \tag{29}$$

with

$$\omega = \frac{2\pi}{24} = \frac{\pi}{12} \tag{30}$$

that is a diurnal variation of the parameters is expressed provided that the sampling interval is one hour.

For the parameters $a_1$ and $\beta_{*,*}$ in equations (29), the following estimates are used (obtained from data from the district heating system in Esbjerg, Denmark): $a_1 = -0.56$, $\beta_{2,0} = 0.35$, $\beta_{2,1} = 0.32$, $\beta_{2,2} = -0.24$, $\beta_{3,0} = 0.18$, $\beta_{3,1} = -0.31$, $\beta_{3,2} = 0.25$, $\beta_{4,0} = 0.18$, $\beta_{4,1} = 0.09$, $\beta_{4,2} = -0.25$ and $\sigma_e^2 = 1.0$.

*Remark*

The above ARMAX model can, in some sense, be interpreted by physical terms. The $A(q^{-1})$ polynomial represents the dynamic behaviour of the system, that is the time constant, which is induced by the heat capacity of the district heating water, the pipes and the surrounding media. The expression $\{1 - B(1)/A(1)\}$ gives the relative heat loss between two points in the network and the time delay $k$ is mainly due to the time that it takes to transport the water from one point to another.

This explains why $a_1$ is constant and $b_{l,t}$ are time varying, viz. the heat capacity is constant, the mass is always the same, but the relative heat loss is time varying, due to varying ambient temperatures and varying heat consumption. Of course, the time delay, $k$, is also time varying [see reference (**3**)], but in this simulation study it is assumed constant.

*5.1.2 Costing horizons*

As the time delay $k$ is assumed known and constant, the minimum costing horizon [see equation (6)] is set to $N_1 = k = 2$. The maximum costing horizon shall at least exceed the degree of $B(q^{-1})$, in this case 4, but reference (**7**) suggested a rather larger value of $N_2$; hence it is set to $N_2 = 10$.

*5.1.3 Penalty matrix*

$\Lambda_t = \lambda\mathbf{I}$, where $\mathbf{I}$ is the identity matrix.

*5.1.4 Reference temperature*

It is assumed that the ambient air temperature is a great deal right of the knee in the minimum curve shown in Fig. 2; hence the influence of the uncertainty of the prediction of the ambient air temperature is eliminated. [The knee is typically located at (10 °C, 70 °C).]

The reference temperature is determined in order to fulfil the constraint in equation (3), that is

$$P\{y_{t+j} \leqslant y^{\min}\} = \alpha, \qquad j = N_1, \ldots, N_2 \tag{31}$$

where $\alpha$ is a service parameter. Since $y_{t+j}$ is a sum of a prediction and prediction error, $y_{t+j} = \hat{y}_{t+j|t} + \tilde{y}_{t+j|t}$, $y^{\text{ref}}_{t+j|t}$ is found by inserting this into equation (31) and then setting $\hat{y}_{t+j|t} = y^{\text{ref}}_{t+j|t}$:

$$P\{y^{\text{ref}}_{t+j|t} + \tilde{y}_{t+j|t} \leqslant y^{\min}\} = \alpha, \qquad j = N_1, \ldots, N_2 \tag{32}$$

Assuming that $\tilde{y}_{t+j|t}$ is normal distributed white noise and that the predictions are computed as a conditional expectation, as described before, it is found that

$$y^{\text{ref}}_{t+j|t} = y^{\min} + \pi_{1-\alpha}\sigma_j, \qquad j = N_1, \ldots, N_2 \tag{33}$$

where $\pi_{1-\alpha}$ is the $100(1-\alpha)$ per cent quantile in the standardized normal distribution and $\sigma_j^2$ is the variance of the $j$-step prediction error. In a stochastic sense the model used is an AR(1) model, and in reference (**10**) it is verified that

$$\sigma_j^2 = \sigma_e^2 \sum_{i=0}^{j-1} a_1^{2i}, \qquad j = N_1, \ldots, N_2 \tag{34}$$

$\alpha = 0.01$ is used in the main study, but the influence of various values of $\alpha$ was also examined. The minimum supply temperature is set to $y^{\min} = 70$ °C.

*5.1.5 Number of time steps*

The simulations are performed over 1000 time steps (hours), but when computing variances, means, maximum, minimum and percentage the first and last 50 time steps are dismissed.

### 5.2 Results

The purpose of the simulation study is threefold: to investigate the effect of different control horizons $NU$, different penalty parameters $\lambda$ and different service parameters $\alpha$.

*5.2.1 $NU$ and $\lambda$ ($\alpha = 0.01$)*

In Fig. 3 the relationships between the control change variance, $\hat{\sigma}^2_{\Delta u}$, and the output variance, $\hat{\sigma}^2_y$, are shown for various values of the penalty parameter, $\lambda$, for $NU = 2$ and 3 respectively. For $\lambda = 0$, $\hat{\sigma}^2_y$ is at minimum, for both $NU = 2$ and 3. As expected, $\hat{\sigma}^2_{\Delta u}$

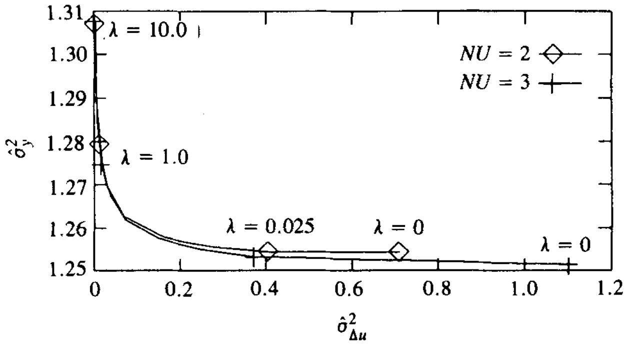


**Fig. 3** Variance of the output signal, $\hat{\sigma}^2_y$, versus the variance of the change in the control signal, $\hat{\sigma}^2_{\Delta u}$, for $NU$ equal to 2 and 3 and for various values of $\lambda$, with $\alpha = 0.01$

decreases and $\hat{\sigma}^2_y$ increases as $\lambda$ is increased. However, for small values of $\lambda$, $\hat{\sigma}^2_y$ is not affected very much while $\hat{\sigma}^2_{\Delta u}$ decreases drastically. For $\lambda \to \infty$ the price of the control effort will be so high that there will be no control at all.

The difference between $NU = 2$ and $NU = 3$ is clearly seen for $\lambda = 0$, but the discrepancy vanishes for higher values of $\lambda$. The results for $NU = 4$ are very similar to those for $NU = 3$; hence, they are not included in the figure. For $NU = 1$ the results would be concentrated in the upper left corner of the figure. For $\lambda = 0$ and $NU = 1$ the results are nearly the same as for $\lambda = 10$ and $NU = 2$ or 3; that is for $NU = 1$ the controller is extremely passive, $\hat{\sigma}^2_{\Delta u}$ is low. This is due to the constraint of the GPC controller expressed in equation (25), that is the future control values are all equal to the present one, $u_t$. The control performance, $\hat{\sigma}^2_y$, for $NU = 1$ is also very poor.

Table 1 shows how various values of $NU$ and $\lambda$ influence the percentage of time below the minimum curve, and the minimum, $\Delta u_{\min}$, the the maximum, $\Delta u_{\max}$, changes in the control signal. It is seen that the percentage compares favourably with the value of the service parameter, $\alpha$, which in this case is set to 0.01. This compares in fact exactly for $NU = 3$ and 4 with $\lambda = 0.0$. As expected, the percentage increases for increasing values of $\lambda$, though these increases are not very high.

In district heating plants, how fast the supply temperature from the plant can be changed is limited. For the district heating system in Esbjerg these limits are $\Delta u_{\max} = 5\,°C$ per hour (when raising the temperature) and $\Delta u_{\min} = -2\,°C$ per hour (when lowering the temperature). Table 1 shows that these constraints are exceeded for $NU = 2$, 3 and 4 and $\lambda = 0.0$. However, this can be coped with by tuning the penalty parameter $\lambda$.

**Table 1** The percentage of the time below the minimum curve and minimum and maximum changes in the control signal for various values of $NU$ and $\lambda$

| $NU$ | $\lambda$ | Percentage | $\Delta u_{\min}$ | $\Delta u_{\max}$ |
|---|---|---|---|---|
| 1 | 0.0 | 1.1 | −0.25 | 0.24 |
| | 0.025 | 1.1 | −0.24 | 0.24 |
| | 1.0 | 1.1 | −0.22 | 0.21 |
| | 10.0 | 1.2 | −0.12 | 0.11 |
| 2 | 0.0 | 1.1 | −3.16 | 3.72 |
| | 0.025 | 1.2 | −1.99 | 2.50 |
| | 1.0 | 1.3 | −0.36 | 0.34 |
| | 10.0 | 1.1 | −0.11 | 0.10 |
| 3 | 0.0 | 1.0 | −4.64 | 4.59 |
| | 0.025 | 1.2 | −1.91 | 2.25 |
| | 1.0 | 1.3 | −0.42 | 0.37 |
| | 10.0 | 1.1 | −0.11 | 0.10 |
| 4 | 0.0 | 1.0 | −4.67 | 4.74 |
| | 0.025 | 1.1 | −2.00 | 2.33 |
| | 1.0 | 1.3 | −0.43 | 0.38 |
| | 10.0 | 1.1 | −0.12 | 0.10 |

**Table 2** The percentage of the time below the minimum curve, the mean of the output, $\bar{y}$, and the input, $\bar{u}$, for different $\alpha$. $NU = 3$ and $\lambda = 0.0$

| $\alpha$ | Percentage | $\bar{y}$ | $\bar{u}$ |
|---|---|---|---|
| 0.0005 | 0.0 | 73.73 | 75.39 |
| 0.001 | 0.11 | 73.50 | 75.15 |
| 0.005 | 0.7 | 72.90 | 74.54 |
| 0.01 | 1.0 | 72.61 | 74.24 |
| 0.02 | 2.2 | 72.29 | 73.92 |
| 0.03 | 3.1 | 72.09 | 73.71 |
| 0.04 | 3.8 | 71.94 | 73.56 |
| 0.05 | 4.4 | 71.82 | 73.43 |

### 5.2.2 *$\alpha$ ($NU = 3$ and $\lambda = 0.0$)*

The only effect that the service parameter $\alpha$ has on the results is that the mean of the output and the input are shifted. Hence the percentage of the time below the minimum curve is changed.

Table 2 contains these results for different $\alpha$ values. It is seen that again the percentage compares favourably with the $\alpha$ values. As expected, the $\bar{y}$ and $\bar{u}$ increase for decreasing values of $\alpha$. The interesting point is to see how much can be gained by choosing a reasonable $\alpha$. As mentioned in the introduction, the saving potentials lie in lowering the supply temperature from the plant. It can, for example, be seen that changing $\alpha$ from 0.01 to 0.03 results in a 0.5 °C lower supply temperature from the plant, on average.

## 6 CONCLUSION

The following conclusions can be drawn based on the simulation results presented:

1. The proposed modified GPC controller is well suited for controlling the supply temperature in district heating systems.
2. By increasing the control horizon, $NU$, the control becomes more active for low values of $\lambda$.
3. It turns out that the difference between various values of $NU$ vanishes for higher $\lambda$.
4. A controller with $NU = 1$ is not interesting.
5. Violations of the $\Delta u$ constraint can be coped with by tuning the penalty parameter $\lambda$.
6. The 'best' controller is obtained with $NU = 2$, 3 or 4 and for small non-zero $\lambda$ values.
7. Higher savings can be achieved by increasing the service parameter $\alpha$, but this can result in more frequent complaints from the consumers.

Implementation of a predictive control (however, not exactly like the one presented here) in the district heating system in Esbjerg, Denmark, has produced considerable energy savings, [see reference (**1**)].

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