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Brief Paper

Generalized Predictive Control for Non-stationary Systems*

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Key Words—Generalized predictive control (GPC); time-varying systems; impulse response; filtering.

Abstract—This paper shows how the generalized predictive control (GPC) can be extended to non-stationary (time-varying) systems. If the time-variation is slow, then the classical GPC can be used in context with an adaptive estimation procedure of a time-invariant ARIMAX model. However, in this paper prior knowledge concerning the nature of the parameter variations is assumed available. The GPC is based on the assumption that the prediction of the system output can be expressed as a linear combination of present and future controls. Since the Diophantine equation *cannot* be used due to the time-variation of the parameters, the optimal prediction is found as the general conditional expectation of the system output.

The underlying model is of an ARMAX-type instead of an ARIMAX-type as in the original version of the GPC (Clarke, D. W., C. Mohtadi and P. S. Tuffs (1987). *Automatica*, **23**, 137–148) and almost all later references. This implies some further modifications of the classical GPC.

1. *Introduction*

MOST FREQUENTLY, when generalized predictive control (GPC) is used for time-varying systems, an adaptive estimation procedure is used for an ordinary ARIMAX model with constant parameters. Hence, the model parameters are assumed to be constant over the prediction horizon applied. The predictions are then, beneficially, obtained by solving the Diophantine equation, e.g. recursively as in Clarke *et al.* (1987). This procedure is reasonable only if the underlying time-variation is relatively slow. In general, it is more reasonble to consider time-varying models and then extend the GPC to handle these models. This allows for control of both slow and fast changing systems, and the time-variation can be used for an improved prediction, and hence for an improved control.

In this paper a GPC for time-varying systems is proposed. This procedure allows for control of systems where, for instance, the parameter variations are so fast that the assumption of the parameters being constant over the prediction horizon is no longer valid. In that case the Diophantine equation is useless. Instead the prediction of the future output is computed directly as the conditional expectations, conditioned on known observations.

In the literature, see e.g. Clarke *et al.* (1987) and Bitmead *et al.* (1990), the GPC is formulated by using an ARIMAX-type model. One of the main arguments for the integrating factor (integral part) is that it guarantees an offset free control, but it also gives a more straightforward formulation of the optimization problem. However, from a model building viewpoint this integral part may seem to be somewhat artificial. Therefore, in this paper the GPC is based on an ARMAX model rather than an ARIMAX model, and the offset control is guaranteed by filtering the control signal.

The paper is organized as follows. In Section 2 the model structure is described and in Section 3 the optimal prediction for non-stationary systems is derived. Section 4 deals with the cost function and the optimization problem. In Section 5 some simulation experiments are presented to illustrate the performance of the controller, and finally the conclusions are drawn in Section 6.

2. *Model structure*

It is assumed that the system can be described by a time-varying ARMAX model

$$A_t(q^{-1})y_t = B_t(q^{-1})u_t + C_t(q^{-1})e_t, \tag{1}$$

where y_t and u_t are the output and the control signal, respectively, e_t is white noise with mean zero and variance σ_e^2 and A_t, B_t and C_t are polynomials in q^{-1} (the back shift operator) with time-varying coefficients:

$$\begin{aligned} A_t(q^{-1}) &= 1 + a_{1,t}q^{-1} + \cdots + a_{n_A,t}q^{-n_A} \\ B_t(q^{-1}) &= b_{1,t}q^{-1} + \cdots + b_{b_B,t}q^{-n_B} \\ C_t(q^{-1}) &= 1 + c_{1,t}q^{-1} + \cdots + c_{n_C,t}q^{-n_C}. \end{aligned} \tag{2}$$

The nature of the parameter variations are assumed known. Consider, as an example, the periodical variation

$$\begin{aligned} a_{j,t} &= \alpha_{j,0} + \alpha_{j,1}\sin(\omega(t-j)) + \alpha_{j,2}\cos(\omega(t-j)) \\ b_{j,t} &= \beta_{j,0} + \beta_{j,1}\sin(\omega(t-j)) + \beta_{j,2}\cos(\omega(t-j)) \\ c_{j,t} &= \gamma_{j,0} + \gamma_{j,1}\sin(\omega(t-j)) + \gamma_{j,2}\cos(\omega(t-j)), \end{aligned} \tag{3}$$

which could, e.g., represent the dirunal variation in an energy system, see e.g. Madsen *et al.* (1992). The parameters α, β and γ may also be time-varying and then they can be estimated adaptively using recursive methods.

3. *Output prediction*

The GPC is based on the assumption that the output predictions can be expressed as a linear combination of present and future controls. In Clarke *et al.* (1987) and many other references this is obtained by solving the Diophantine equation (sometimes recursively). However, in the time-varying case the Diophantine equation *cannot* be used due to

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the time-variation of the model parameters. Instead, the j-step predictor, $\hat{y}_{t+j|t}$, is found as the conditional expectation of y_{t+j} conditioned on observations of the output up to time t (Madsen, 1989)

$$\hat{y}_{t+j|t} = -\sum_{i=1}^{n_A} a_{i,t+j}\hat{y}_{t+j-i|t} + \sum_{i=1}^{n_B} b_{i,t+j}u_{t+j-i} + \sum_{i=0}^{n_C} c_{i,t+j}\hat{e}_{t+j-i|t}, \quad j \geq 1 \tag{4}$$

$$\hat{y}_{t+j|t} = y_{t+j}, \quad j < 1, \tag{5}$$

where

$$\hat{e}_{t+l|t} = \begin{cases} 0, & \text{if } l \geq 1 \\ e_{t+l}, & \text{if } l < 1. \end{cases} \tag{6}$$

A simple example illustrates the method.

Example. Consider the ARX model [equations (1) and (2)] ($n_A = 1, n_B = 2, n_C = 0$)

$$y_t + a_{1,t}y_{t-1} = b_{1,t}u_{t-1} + b_{2,t}u_{t-2} + e_t. \tag{7}$$

The 1-step predictor is derived from equations (4)–(6) as

$$\begin{aligned}\hat{y}_{t+1|t} &= -a_{1,t+1}y_t + b_{1,t+1}u_t + b_{2,t+1}u_{t-1} \\ &= h_{1,t+1}u_t + v_{1,t},\end{aligned} \tag{8}$$

where $v_{1,t} = -a_{1,t+1}y_t + b_{2,t+1}u_{t-1}$ is known and $h_{1,t+1}u_t = b_{1,t+1}u_t$ is unknown until the control signal is chosen at time t. Note, that $h_{1,t+1} = b_{1,t+1}$ is the first weight of the time-varying impulse response function.

In general, the j-step predictor is given as

$$\begin{aligned}\hat{y}_{t+j|t} &= -a_{1,t+j}\hat{y}_{t+j-1|t} + b_{1,t+j}u_{t+j-1} + b_{2,t+j}u_{t+j-2} \\ &= -a_{1,t+j}(h_{j-1,t+j-1}u_t + \cdots + h_{1,t+j-1}u_{t+j-2} \\ &\quad + v_{j-1,t}) + b_{1,t+j}u_{t+j-1} + b_{2,t+j}u_{t+j-2} \\ &= h_{j,t+j}u_t + \cdots + h_{1,t+j}u_{t+j-1} + v_{j,t} \\ &= \sum_{i=1}^{j} h_{i,t+j}u_{t+j-i} + v_{j,t}.\end{aligned} \tag{9}$$

For the given model the $h_{i,t}$ and $v_{i,t}$ values can be computed recursively as

$$h_{i,t} = \begin{cases} b_{i,t}, & \text{if } i = 1 \\ b_{i,t} - a_{1,t}h_{i-1,t-1}, & \text{if } i = 2 \\ -a_{1,t}h_{i-1,t-1} & \text{if } i \geq 3 \end{cases} \tag{10}$$

and

$$v_{i,t} = \begin{cases} -a_{1,t+i}y_t + b_{2,t+i}u_{t-1}, & \text{if } i = 1 \\ -a_{1,t+i}v_{i-1,t}, & \text{if } i \geq 2. \end{cases} \tag{11}$$

The coefficients $h_{i,t+j}$ $(i = 1, 2, \ldots)$ are the weights of the time-varying impulse response function describing the dynamic relation between the input and the output, i.e. $h_{i,t}$ is the marginal change of y_t changing u_{t-i}. □

It can be shown that a j-step predictor for an arbitrary ARMAX process can be written in the form shown in equation (9). This can, for instance, be shown by substitution as illustrated in the previous example.

An alternative scheme for calculating $h_{i,t+j}$ and $v_{j,t}$ follows directly from equation (9):

(1) to find $v_{j,t}$, set the present and future control to zero ($u_{t+j-i} = 0$ for $i = 1, 2, \ldots, j$). Then compute $v_{j,t}$ as the conditional expectation, $\hat{y}_{t+j|t}$, using equation (4); and

(2) to find $h_{i,t+j}$ $(i = 1, 2, \ldots, j)$, set the present and past output and past control to zero ($v_{j,t} = 0$). Feed an impulse into the system at time $t + j - i$

$$u_{t+j-l} = \begin{cases} 1, & \text{if } l = i \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

Then compute $h_{i,t+j}$ as the conditional expectation, $\hat{y}_{t+j|t}$, using equation (4).

Now introduce a maximum prediction horizon $N(\geq 1)$. Considering equation (9) it is seen that the j-step predictions, j running from 1 up to N, can be written as a linear matrix expression:

$$\hat{\mathbf{y}}_t = \mathbf{H}_t\mathbf{u}_t + \mathbf{v}_t, \tag{13}$$

where

$$\hat{\mathbf{y}}_t = [\hat{y}_{t+1|t}, \ldots, \hat{y}_{t+N|t}]^T$$
$$\mathbf{u}_t = [u_t, \ldots, u_{t+N-1}]^T$$
$$\mathbf{v}_t = [v_{1,t}, \ldots, v_{N,t}]^T$$
$$\mathbf{H}_t = \begin{bmatrix} h_{1,t+1} & 0 & \cdots & 0 & 0 \\ h_{2,t+2} & h_{1,t+2} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ h_{N-1,t+N-1} & h_{N-2,t+N-1} & \cdots & h_{1,t+N-1} & 0 \\ h_{N,t+N} & h_{N-1,t+N} & \cdots & h_{2,t+N} & h_{1,t+N} \end{bmatrix}.$$

In equation (13) it has been assumed that the same model is applied for all prediction horizons. Actually, this need not be the case. If different models are used, then the jth row of $\mathbf{H}_t$ and the jth element of $\mathbf{v}_t$ belongs to a special model designed for j-step prediction. Making use of an individual model for each horizon is often relevant if a non-linear system is approximated by a family of linear models (e.g. threshold models), that is if the optimal linearization of the system depends on the prediction horizon.

4. *Cost function and optimization*

Consider a cost function of the form

$$J = E\left[\sum_{j=N_1}^{N_2} (y_{t+j} - y_{t+j|t}^{\text{ref}})^2 + \sum_{j=1}^{NU} \lambda_{j,t}u_{t+j-1}^2\right], \tag{14}$$

where N_1 is the minimum costing horizon, N_2 is the maximum costing horizon, $\lambda_{j,t}$ is a control-weighting (or penalty) sequence, NU is the control horizon and $y_{t+j|t}^{\text{ref}}$ is the future reference output. The expectation in equation (14) is conditioned on observations available at time t.

The cost function expressed in equation (14) is almost identical to the original GPC cost function presented in Clarke *et al.* (1987). The only exception is that in Clarke *et al.* (1987) the control increments, Δu_t, are used instead of the absolute control values, u_t. The incremental version of the cost function will be discussed later.

Introducing matrix notation the cost function is written as

$$J = E[(\mathbf{y}_t - \mathbf{y}_t^{\text{ref}})^T(\mathbf{y}_t - \mathbf{y}_t^{\text{ref}}) + \mathbf{u}_t^T\mathbf{\Lambda}_t\mathbf{u}_t], \tag{15}$$

where

$$\mathbf{y}_t = [y_{t+N_1}, \ldots, y_{t+N_2}]^T$$
$$\mathbf{y}_t^{\text{ref}} = [y_{t+N_1|t}^{\text{ref}}, \ldots, y_{t+N_2|t}^{\text{ref}}]^T$$
$$\mathbf{u}_t = [u_t, \ldots, u_{t+NU-1}]^T$$
$$\mathbf{\Lambda}_t = \begin{bmatrix} \lambda_{1,t} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_{NU,t} \end{bmatrix}.$$

By the projection theorem (Brockwell and Davis, 1987) the output vector, $\mathbf{y}_t$, is decomposed into two (stochastic) independent parts

$$\mathbf{y}_t = \hat{\mathbf{y}}_t + \tilde{\mathbf{y}}_t, \tag{16}$$

where $\hat{\mathbf{y}}_t$ is a vector containing the output predictions described in the previous section, and $\tilde{\mathbf{y}}_t$ is a vector containing the prediction errors

$$\tilde{\mathbf{y}}_t = [\tilde{y}_{t+N_1|t}, \ldots, \tilde{y}_{t+N_2|t}]^T. \tag{17}$$

The cost function can then be written as

$$J = [(\hat{\mathbf{y}}_t - \mathbf{y}_t^{\text{ref}})^T(\hat{\mathbf{y}}_t - \mathbf{y}_t^{\text{ref}}) + \mathbf{u}_t^T\mathbf{\Lambda}_t\mathbf{u}_t] + E[\tilde{\mathbf{y}}_t^T\tilde{\mathbf{y}}_t]. \tag{18}$$

Note that the last term in this expression does not depend on

$\mathbf{u}_t$. Using equation (13)* the cost function becomes

$$J = [(\mathbf{H}_t\mathbf{u}_t + \mathbf{v}_t - \mathbf{y}_t^{\text{ref}})^T(\mathbf{H}_t\mathbf{u}_t + \mathbf{v}_t - \mathbf{y}_t^{\text{ref}}) + \mathbf{u}_t^T\mathbf{\Lambda}_t\mathbf{u}_t] + E[\tilde{\mathbf{y}}_t^T\tilde{\mathbf{y}}_t]. \quad (19)$$

The GPC control law is then obtained by minimizing the resulting cost function. This is done by setting the derivative of the cost function with respect to $\mathbf{u}_t$ to zero, i.e.

$$\frac{\partial J}{\partial \mathbf{u}_t} = 0.$$

That is

$$2\mathbf{H}_t^T(\mathbf{H}_t\mathbf{u}_t + \mathbf{v}_t - \mathbf{y}_t^{\text{ref}}) + 2\mathbf{\Lambda}_t\mathbf{u}_t = 0 \quad (20)$$

or

$$2(\mathbf{H}_t^T\mathbf{H}_t + \mathbf{\Lambda}_t)\mathbf{u}_t + 2\mathbf{H}_t^T(\mathbf{v}_t - \mathbf{y}_t^{\text{ref}}) = 0. \quad (21)$$

Solving for $\mathbf{u}_t$, results in

$$\mathbf{u}_t = -[\mathbf{H}_t^T\mathbf{H}_t + \mathbf{\Lambda}_t]^{-1}\mathbf{H}_t^T(\mathbf{v}_t - \mathbf{y}_t^{\text{ref}}). \quad (22)$$

Only the first element of the control vector, $\mathbf{u}_t$, is implemented (recording horizon control) so the control law can be written as

$$u_t = -\mathbf{\theta}[\mathbf{H}_t^T\mathbf{H}_t + \mathbf{\Lambda}_t]^{-1}\mathbf{H}_t^T(\mathbf{v}_t - \mathbf{y}_t^{\text{ref}}), \quad (23)$$

where

$$\mathbf{\theta} = [1, 0, \ldots, 0]. \quad (24)$$

4.1. *The filtered version.* The control law in equation (23) may give an offset for $\mathbf{\Lambda}_t > \mathbf{0}$. This is coped with by introducing a filtered version of the control signal in the cost function.

The filter is defined as

$$\bar{u}_t = \frac{P_N(q^{-1})}{P_D(q^{-1})}u_t, \quad (25)$$

where $P_N(q^{-1})$ and $P_D(q^{-1})$ are polynomials in the back shift operator q^{-1}.

The filtered control signal can be decomposed into a sum of two terms: one term containing past control signals, and another containing the present and future control signals. This can be done by means of solving the Diophantine equation (since the filter is time-invariant).

$$\bar{u}_{t+j} = \frac{D_j(q^{-1})}{P_D(q^{-1})}u_t + E_j(q^{-1})u_{t+j}. \quad (26)$$

By introducing matrix notation this can be rewritten as

$$\bar{\mathbf{u}}_t = \mathbf{F}\mathbf{u}_t + \mathbf{g}_t, \quad (27)$$

where

$$\bar{\mathbf{u}}_t = [\bar{u}_t, \ldots, \bar{u}_{t+NU-1}]^T$$
$$\mathbf{u}_t = [u_t, \ldots, u_{t+NU-1}]^T$$
$$\mathbf{g}_t = \bar{\mathbf{u}}_t|_{\mathbf{u}_t=\mathbf{0}}$$
$$\mathbf{F} = \begin{bmatrix} f_1 & 0 & \cdots & 0 \\ f_2 & f_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ f_{NU} & f_{NU-1} & \cdots & f_1 \end{bmatrix}.$$

$\mathbf{F}$ contains the impulse response weights for the filter.

Using the filtered control signal equation (27) instead of the unfiltered in the cost function, equation (15), leads to

$$J = [(\mathbf{H}_t\mathbf{u}_t + \mathbf{v}_t - \mathbf{y}_t^{\text{ref}})^T(\mathbf{H}_t\mathbf{u}_t + \mathbf{v}_t - \mathbf{y}_t^{\text{ref}}) + (\mathbf{F}\mathbf{u}_t + \mathbf{g}_t)^T\mathbf{\Lambda}_t(\mathbf{F}\mathbf{u}_t + \mathbf{g}_t)] + E[\tilde{\mathbf{y}}_t^T\tilde{\mathbf{y}}_t]. \quad (28)$$

Minimization of this expression results in the control vector

$$\mathbf{u}_t = -[\mathbf{H}_t^T\mathbf{H}_t + \mathbf{F}^T\mathbf{\Lambda}_t\mathbf{F}]^{-1}[\mathbf{H}_t^T(\mathbf{v}_t - \mathbf{y}_t^{\text{ref}}) + \mathbf{F}^T\mathbf{\Lambda}_t\mathbf{g}_t] \quad (29)$$

and the implemented control at time t

$$u_t = -\mathbf{\theta}[\mathbf{H}_t^T\mathbf{H}_t + \mathbf{F}^T\mathbf{\Lambda}_t\mathbf{F}]^{-1}[\mathbf{H}_t^T(\mathbf{v}_t - \mathbf{y}_t^{\text{ref}}) + \mathbf{F}^T\mathbf{\Lambda}_t\mathbf{g}_t] \quad (30)$$

with $\mathbf{\theta}$ as in equation (24).

For the important special case $\bar{u}_t = \Delta u_t = u_t - u_{t-1}$, $\mathbf{F}$ and $\mathbf{g}_t$ in equation (27) are given by

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ -1 & 1 & \cdots & 0 & 0 \\ 0 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -1 & 1 \end{bmatrix} \quad (31)$$

and

$$\mathbf{g}_t = [-u_{t-1}, 0, \ldots, 0]^T. \quad (32)$$

Remark. The main purpose of introducing the filter is to eliminate the offset, induced by the cost function in equation (14) for positive penalties, since the underlying model does not have an integration.

The filter choice depends on which variations in the control signal should be penalized. For instance the high frequent variations are penalized by using the incremental filter as defined by $\mathbf{F}$ and $\mathbf{g}$ in equations (31) and (32). The steady-state gain of the chosen filter should be equal to zero (in order to eliminate the offset).

5. *Simulation experiments*

For the simulation studies the following ARX model is used

$$y_t = -a_1 y_{t-1} + b_{1,t}u_{t-2} + b_{2,t}u_{t-3} + b_{3,t}u_{t-4} + e_t, \quad (33)$$

where a_1 is constant and $b_{l,t}$ ($l = 2, 3, 4$) are given by

$$b_{2,t} = \beta_{2,0} + \beta_{2,1}\sin(\omega(t-2)) + \beta_{2,2}\cos(\omega(t-2))$$
$$b_{3,t} = \beta_{3,0} + \beta_{3,1}\sin(\omega(t-3)) + \beta_{3,2}\cos(\omega(t-3))$$
$$b_{4,t} = \beta_{4,0} + \beta_{4,1}\sin(\omega(t-4)) + \beta_{4,2}\cos(\omega(t-4)),$$

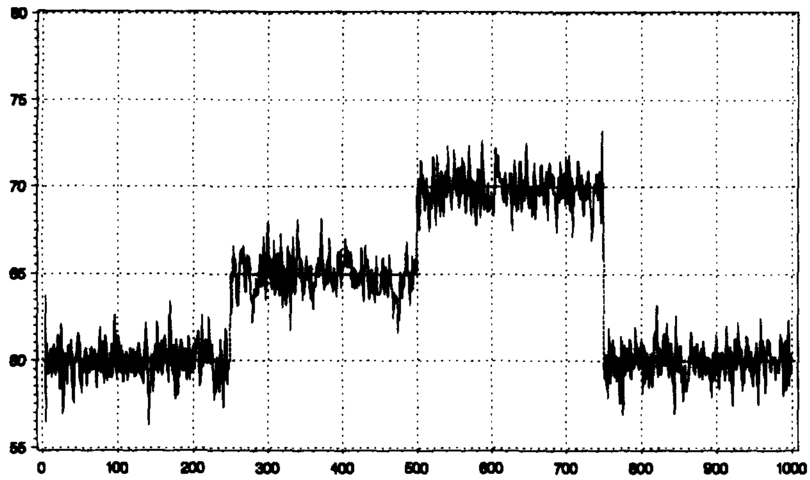
with $\omega = 2\pi/24 = \pi/12$. The following values are used: $a_1 = -0.56$, $\beta_{2,0} = 0.35$, $\beta_{2,1} = 0.32$, $\beta_{2,2} = -0.24$, $\beta_{3,0} = 0.18$, $\beta_{3,1} = -0.31$, $\beta_{3,2} = 0.25$, $\beta_{4,0} = 0.18$, $\beta_{4,1} = 0.09$, $\beta_{4,2} = -0.25$, and $e_t \sim N(0, 1)$. The horizons N_1, N_2 and NU are set to 2, 5 and 2, respectively. $\mathbf{\Lambda}_t$ is assumed constant and $\mathbf{\Lambda}_t = \lambda\mathbf{I}$.

Figures 1 and 2 show the output, the reference output and the control signal for $\lambda = 0.0$ and 0.05, respectively. The incremental filter [equations (31) and (32)] is used. The

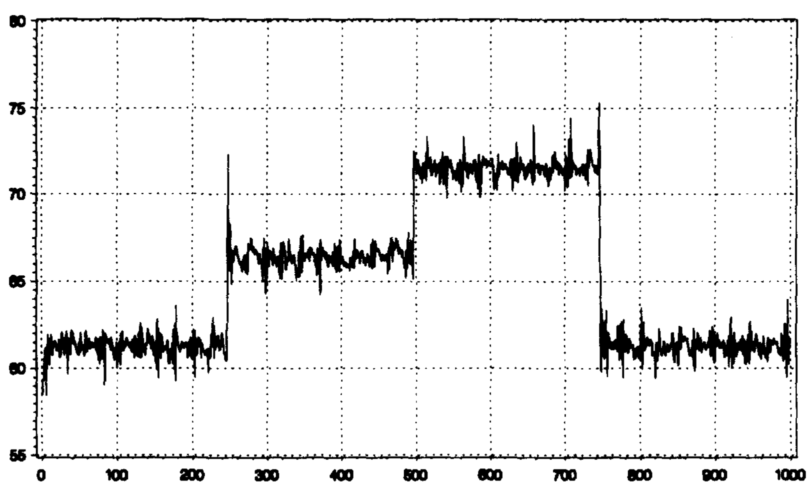
* The time-varying impulse response matrix is now written, according to the horizons N_1, N_2 and NU (assuming that $N_2 > NU + N_1$), as

$$\mathbf{H}_t = \begin{bmatrix} h_{1,t+N_1} & 0 & \cdots & 0 \\ h_{2,t+N_1+1} & h_{1,t+N_1+1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h_{NU,t+N_1+NU-1} & h_{NU-1,t+N_1+NU-1} & \cdots & h_{1,t+N_1+NU-1} \\ h_{NU+1,t+N_1+NU} & h_{NU,t+N_1+NU} & \cdots & h_{2,t+N_1+NU} + h_{1,t+N_1+NU} \\ \vdots & \vdots & & \vdots \\ h_{N_2-N_1+1,t+N_2} & h_{N_2-N_1,t+N_2} & \cdots & \sum_{i=1}^{N_2-N_1-NU+2} h_{i,t+N_2} \end{bmatrix}.$$

The summation in the last column results from the assumption $\Delta u_{t+i-1} = 0, i > NU$ (Bjerre, 1992). The vector $\mathbf{v}_t$ is now $\mathbf{v}_t = [v_{N_1,t}, \ldots, v_{N_2,t}]^T$.



(a)



(b)

FIG. 1. (a) The output y_t and the reference output $y_{y_t}^{ref}$. (b) The control signal u_t, $\lambda = 0.0$.

changes in the reference output, y_t^{ref}, are made in steps as indicated in the figures. Changing λ from 0 to 0.05 reduces the amplitude of the control signal noticeably, without any visible change in the output. This is also illustrated in Figs 3 and 4, which expresses the control performance ($\Sigma_{t=50}(y_t - y_t^{ref})^2$) and the control effort ($\Sigma_{t=50}(\Delta u_t)^2$) as a function of the time t. It is seen that for $\lambda = 0.05$ the control effort is much lesser than for $\lambda = 0.0$, wthout any markable decrease in the control performance.

Figures 5 and 6 show simulations where the knowledge of the explicit time-variations in the model parameters are not used in prediction (13), i.e. when computing the impulse response matrix $\mathbf{H}_t$ and the vector $\mathbf{v}_t$. The same time-varying model with the same parameters is used, but instead of using the knowledge of the explicit time-variations, the parameters are assumed to be constant and equal to their value at time t, i.e. $b_{2,t+j} = b_{2,t}$, $b_{3,t+j} = b_{3,t}$ and $b_{4,t+j} = b_{4,t}$ for $j = N_1, \ldots, N_2$. It is seen that this increases the control effort for $\lambda = 0$, but the control performance is unchanged. Both the control performance and the control effort are nearly the same as before, for $\lambda = 0.05$. This is explained by the fact that the prediction is not as accurate as it is in the previous case (Figs 1–4) and therefore it affects the control increments, Δu_t in a negative way. This effect is deleted when the control increments are penalized, i.e. $\lambda > 0$.

The effect of using an unfiltered control signal is shown in Fig. 7, i.e. the controller expressed in equation (23) is used. If Figs 4 and 7 are compared it is clearly seen, as expected, that if the control signal is not filtered this may result in an offset control for $\lambda > 0$. Obviously there is no difference when $\lambda = 0$ [compare equations (23) and (30) for $\mathbf{\Lambda} = \mathbf{0}$], and therefore this case is not shown.

6. *Conclusion*

In this paper the classical GPC is extended to cope with time-varying systems, where the nature of the parameter variations is assumed known. This is done by using the knowledge of the explicit parameter variations when computing the prediction of the future output as a conditional expectation. This is reflected in the resulting time-varying impulse response matrix.

The GPC is based on an ARMAX model instead of an ARIMAX model as in almost all GPC references. This is advantageous from a model building point of view if the system does not contain an integration. The offset free control is achieved by filtering the control signal.

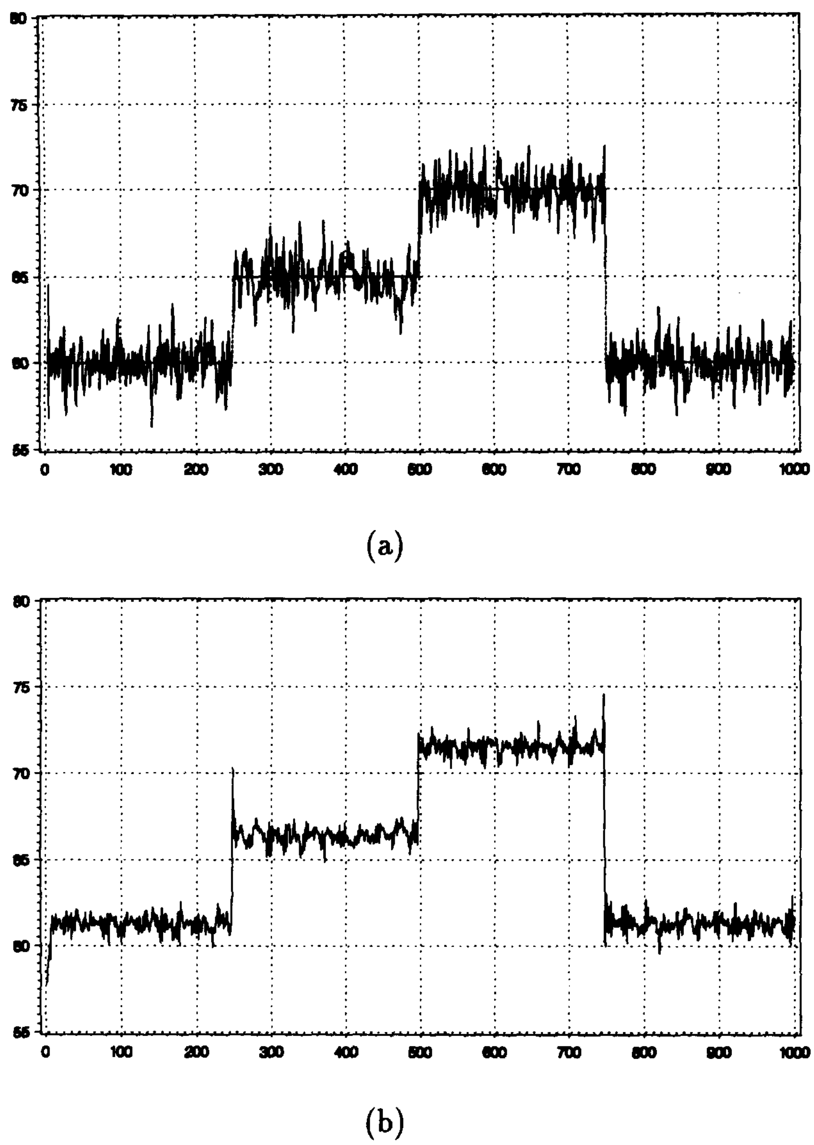


FIG. 2. (a) The output y_t and the reference output y_t^{ref}. (b) The control signal u_t, $\lambda = 0.05$.

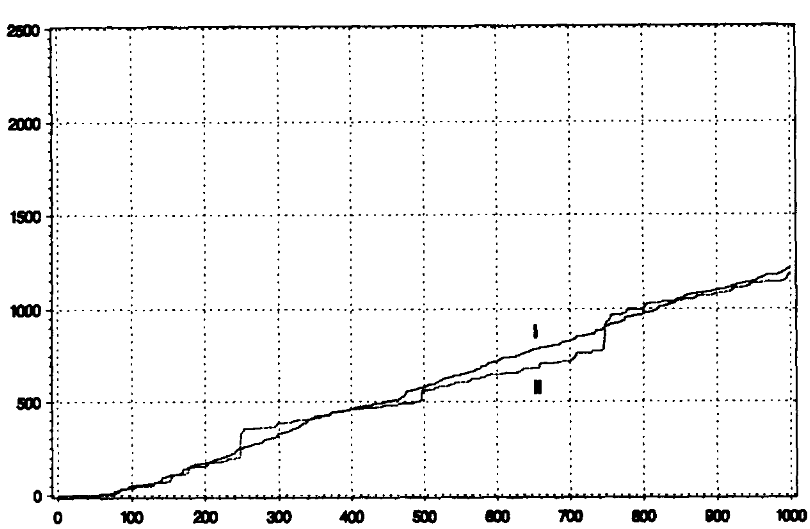


FIG. 3. $\sum_{t=50}(y_t - y_t^{ref})^2$ (I) and $\sum_{t=50}(\Delta u_t)^2$ (II) as a function of the time t, $\lambda = 0$.

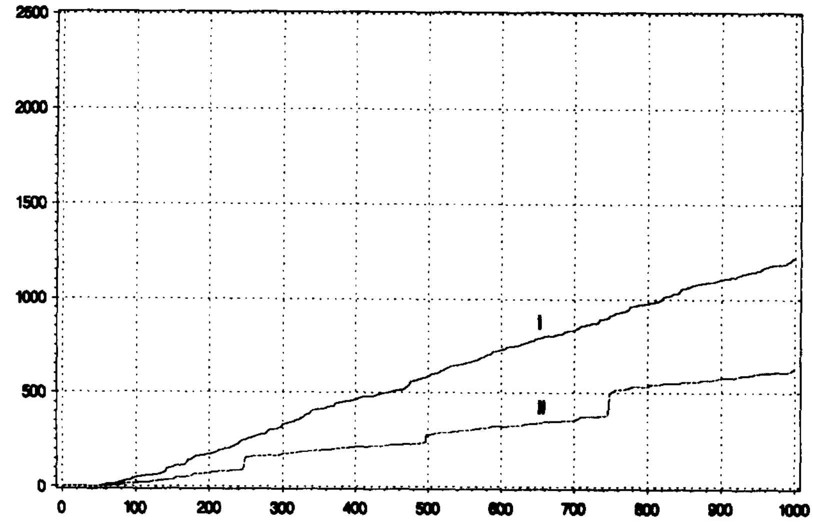


FIG. 4. $\sum_{t=50}(y_t - y_t^{\text{ref}})^2$ (I) and $\sum_{t=50}(\Delta u_t)^2$ (II) as a function of the time t, $\lambda = 0.05$.

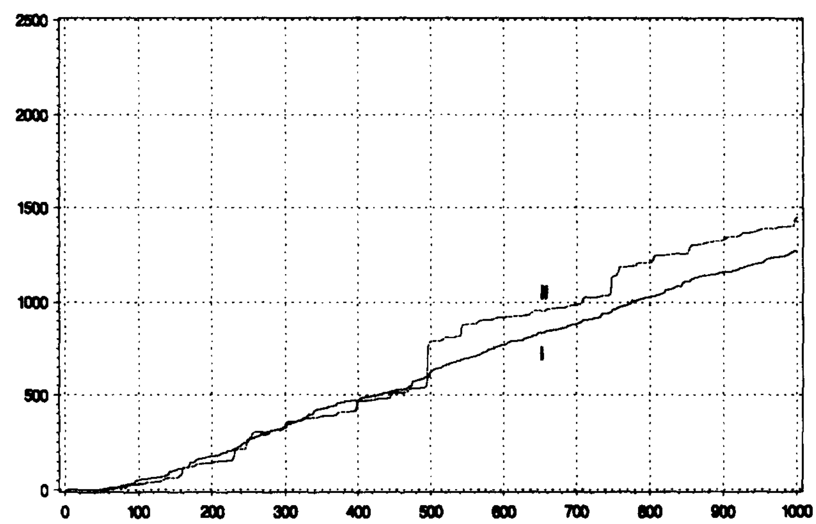


FIG. 5. $\sum_{t=50}(y_t - y_t^{\text{ref}})^2$ (I) and $\sum_{t=50}(\Delta u_t)^2$ (II) as a function of the time t, $\lambda = 0$. The parameters are assumed constant when computing the impulse response matrix.

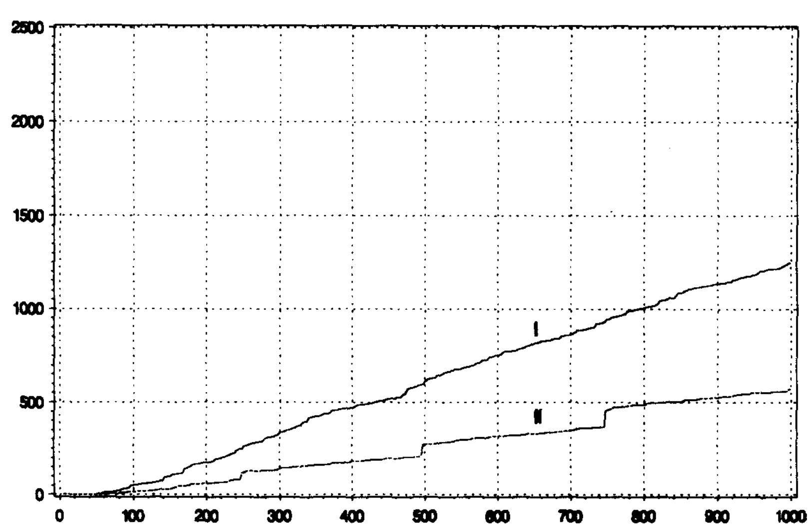


FIG. 6. $\sum_{t=50}(y_t - y_t^{\text{ref}})^2$ (I) and $\sum_{t=50}(\Delta u_t)$ (II) as a function of the time t, $\lambda = 0.05$. The parameters are assumed constant when computing the impulse response matrix.

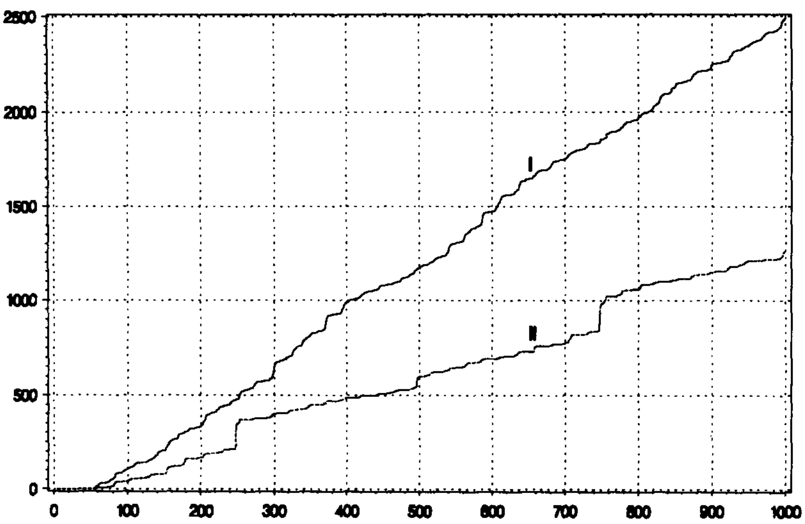


FIG. 7. $\sum_{t=50}(y_t - y_t^{ref})^2$ (I) and $\sum_{t=50}(\Delta u_t)^2$ (II) as a function of the time t, $\lambda = 0.05$. Unfiltered control signal.

A simulation study shows the effect of the abovementioned extensions and modification.

The proposed GPC for time-varying systems keeps all of the advantages of the classical GPC, but the stability and the convergence properties of the proposed GPC must be studied further.

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