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ON FLOW AND SUPPLY TEMPERATURE CONTROL IN DISTRICT HEATING SYSTEMS

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Abstract—This paper discusses how the control of the flow and the supply temperature in district heating systems can be optimized, utilizing stochastic modelling, prediction and control methods. The main objective is to reduce heat production costs and heat losses in the transmission and distribution net by minimizing the supply temperature at the district heating plant. This control strategy is reasonable, in particular, if the heat production takes place at a combined heat and power (CHP) plant. The control strategy is subject to some restrictions, e.g. that the total heat requirement for all consumers is supplied at any time, and each individual consumer is guaranteed some minimum supply temperature at any time. Another important restriction is that the variation in time of the supply temperature is kept as small as possible. This concept has been incorporated in the program package, PRESS, developed at the Technical University of Denmark. PRESS has been applied and tested, e.g. at Vestkraft in Esbjerg, Denmark, and significant saving potentials have been documented. PRESS is now distributed by the Danish District Heating Association.

NOMENCLATURE

a, b	model parameters
c_w	specific heat of the water (J kg^{-1}°C^{-1})
$I_{\text{workday},t}$	indicator function being 1 on workdays and 0 otherwise
j	prediction horizon
l	adjusting parameter accounting for the variables having mean values different from zero (J s^{-1})
p_t	heat load (J s^{-1})
$\hat{p}_{t+1\|t}$	one-step prediction, at time t, of the heat load at time $t+1$ (J s^{-1})
P	probability measure
N_1	minimum horizon
N_2	maximum horizon
N_u	control horizon
q	mass flow (kg s^{-1})
t	time (h)
$T_{a,t}$	ambient air temperature (°C)
$T_{n,t}$	supply temperature in the network (°C)
$T_{r,t}$	return temperature at the plant (°C)
$T_{s,t}$	supply temperature from the plant ($\Delta T_{s,t} = T_{s,t} - T_{s,t-1}$) (°C)
w_t	wind speed (ms^{-1})

Greek letters

α, β	model parameters
γ	weights
ϵ	mean zero model error
π	probability
$\mu_{1,t}$	diurnal heat load profile for workdays (J s^{-1})
$\mu_{2,t}$	diurnal heat load profile for weekends (J s^{-1})

Superscripts and subscripts

a	ambient
max	maximum
n	network
s	supply
ref	reference value
$\hat{}$	prediction
$t+1\|t$	(prediction) at $t+1$ given the information at t
w	water

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1. INTRODUCTION

In district heating systems, where the heat is produced at CHP plants, the simplest and most widely used is the central temperature control method [1]. In this case, the required amount of heat is provided to the consumer by varying the supply temperature at the power plant as a function of the current ambient air temperature. Such functions (control curves) can be found, e.g. in refs. [1–3]; this means that the flow rate of water circulated in the district heating system is almost constant during the heating period [1]. This control strategy must necessarily be very cautious, i.e. frequently the supply temperature can be higher than it has to, since the ambient air temperature is the only meteorological factor considered, and the control curves do not take into account the dynamic characteristic of the district heating system and the diurnal variation of the heat consumption.

The main saving potential in district heating systems, supplied by CHP, lies in lowering the supply temperature from the plant, hence lowering the heat losses in the transmission and the distribution network and lowering the production costs. The savings in the production costs are due to the fact that decreasing the supply temperature implies an increase in the ratio of the power to heat output, and as electricity is more valuable than heat a more profitable operation is achieved [4].

Thus, a reasonable control strategy is to keep the supply temperature from the district heating plant as low as possible, but subject to the following restrictions:

1. The total heat requirement for all consumers is supplied at any time.
2. Each individual consumer is guaranteed some minimum supply temperature at any time. This supply temperature depends on the ambient air temperature.
3. The variation in time of the supply temperature from the district heating plant is kept reasonably small.

The main idea in the proposed control strategy is illustrated schematically in Fig. 1. The first restriction is coped with by using models for prediction of the heat load. An extensive research on predicting heat load has been done, and the main results are found in refs. [2, 5, 6]. By observing the return temperature of the water and assuming maximum water circulation, the lowest admissible supply temperature in the next time step (equal to 1 h in Esbjerg, but other time steps can equally well be applied) is computed from energy balance equations. The uncertainty of the prediction is taken into account in this calculation. A more advanced controller has been proposed in refs. [2, 7]. In Fig. 1 this flow sub-controller is denoted as FSC.

The second restriction is dealt with by finding several representative (or critical) points (locations) in the distribution networks, defined as follows: if the temperature requirements at these points are met, then the temperature requirements are met for all consumers. For each of the representative points a predictive type sub-controller (SC in Fig. 1) is developed in order to compute the lowest supply temperature from the plant, ensuring that the temperature at that particular point is

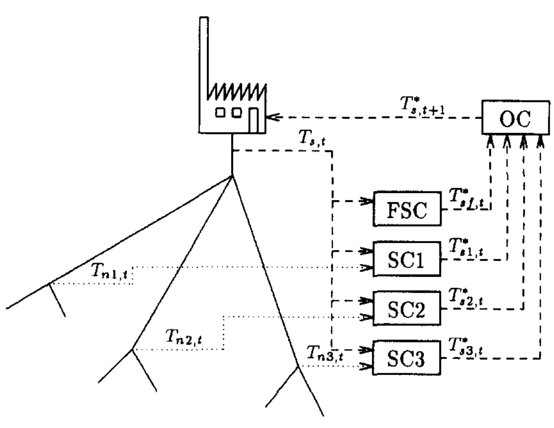

Fig. 1. A sketch of a district heating network, with three representative points and the controllers. OC is the overall controller, FSC is the flow sub-controller, SC are the supply temperature sub-controllers, $T_{n,t}$ are the supply temperatures in the network, $T_{s,t}$ is the supply temperature from the plant, and $T^*_{s,t}$ are the supply temperatures required by the sub-controllers.

minimized, but above an ambient air temperature-dependent minimum. In the computation the time-delay of temperatures between the plant and the point is taken into account. The sub-controller also satisfies the third restriction which is due to the fact that rapidly changing the supply temperature from the plant is not desirable as it implies wearing of the pipelines and causes undesirable operation of the plant.

The overall controller (OC in Fig. 1) selects the highest supply temperature among required supply temperatures calculated from predictions of the heat load and from the above-mentioned sub-controllers for the representative points in the distribution network. This temperature is used as the supply temperature from the plant in the following hour (Fig. 1).

The remainder of this paper is organized as follows: in Section 2 the models for prediction of the heat load are briefly described and how they are used for minimizing the supply temperature is discussed, hence leading to the flow sub-controller. Section 3 is devoted to the supply temperature sub-controller. Section 4 describes the overall controller, and Section 5 gives some practical experience. Finally, some conclusions are drawn in Section 6.

2. THE FLOW SUB-CONTROLLER

2.1. *Prediction of the heat load*

Forecasting the future heat load, using stochastic methods, has been dealt with in refs. [2, 5, 6, 8]. Forecasting of heat load has also been studied in refs. [9, 10], and in ref. [11], where neural networks are applied.

The potential external variables in a model describing the heat load are, e.g. the ambient air temperature, the wind velocity (and direction), humidity/precipitation, the temperature of the supply water, the temperature of the return water, the water flow, the time of day and type of day (weekend and workdays). Various types of models have been considered, e.g. general transfer function models, non-linear transfer function models, smooth threshold transfer function models, and neural network models. The first class contains linear models, whereas the rest are non-linear models.

The models found best suited for control purpose are the general transfer function models, see refs. [5, 6]. These models give rise to the one-step predictor:

$$\begin{aligned}\hat{p}_{t+1|t} = {} & \alpha_1 p_t + \alpha_2 p_{t-1} + \alpha_{24} p_{t-23} + \alpha_{25} p_{t-24} + \alpha_{26} p_{t-25} \\ & + \beta_{1,0}\Delta T_{s,t+1} + \beta_{1,1}\Delta T_{s,t} + \beta_{2,1} T_{a,t} + \beta_{2,2} T_{a,t-1} + \beta_{3,1} w_t + \beta_{3,2} w_{t-1} \\ & + \mu_{1,t+1} I_{\text{workday},t+1} + \mu_{2,t+1}(1 - I_{\text{workday},t+1}) + l. \end{aligned} \tag{1}$$

The predictors proposed for prediction horizons, j, between 2 and 23 h are:

$$\begin{aligned}\hat{p}_{t+j|t} = {} & \alpha_j p_t + \alpha_{j+1} p_{t-1} + \alpha_{24} p_{t+j-24} + \alpha_{25} p_{t+j-25} + \alpha_{26} p_{t+j-26} \\ & + \beta_{1,0}\Delta T_{s,t+j} + \beta_{1,1}\Delta T_{s,t+j-1} + \beta_{1,2}\Delta T_{s,t+j-2} + \beta_{1,3}\Delta T_{s,t+j-3} \\ & + \beta_{2,j} T_{a,t} + \beta_{2,j+1} T_{a,t-1} + \beta_{2,j+2} T_{a,t-2} + \beta_{2,j+3} T_{a,t-3} \\ & + \beta_{3,j} w_t + \beta_{3,j+1} w_{t-1} + \beta_{3,j+2} w_{t-2} + \beta_{3,j+3} w_{t-3} \\ & + \mu_{1,t+j} I_{\text{workday},t+j} + \mu_{2,t+j}(1 - I_{\text{workday},t+j}) + l. \end{aligned} \tag{2}$$

Note that an individual model is set up for each prediction horizon. The parameters are re-estimated each hour.

2.2. *The flow sub-controller*

In the present case the control variable is the supply temperature and the process to be controlled is the mass flow. Actually, the system will not allow the mass flow to exceed some maximum value due to physical limitations, but if it happens that the consumers' demand of mass flow adds up to exceed this maximum, the result will be that the most distant consumers will not have their heat demand fulfilled.

Normally, variations of the heat demand can be met, either by varying the mass flow through the network or by varying the supply temperature. Since the object is to keep the supply temperature as low as possible, the heat demand is met by varying the mass flow as far as possible. If the heat demand is suitably low the supply temperature is kept near its minimum, while the mass flow is subject to variations. For moderately high heat demand the mass flow occasionally reaches its upper limit during the day, implying that an increase of the temperature is needed. If the amount of heat required by the consumers becomes so large that the mass flow is around its maximum throughout the day (which is usually the case for non-summer situations), variation of the temperature is the only way to meet varying demands.

Since the process to be controlled is the mass flow, a conversion from predicted heat load to predicted water flow has to be carried out. In this conversion future supply and return temperatures are used.

2.2.1. *The reference mass flow.* Suppose that future values of the mass flow, q_{t+j}, have to be below the maximum flow, $q^{\max}$, with probability π (e.g. 99%), then

$$P\{q_{t+j} \leqslant q^{\max}\} = \pi, \quad 0 < N_1 \leqslant j \leqslant N_2, \tag{3}$$

where N_1 and N_2 are chosen such that $q_{t+N_1}, \ldots, q_{t+N_2}$ encompass future mass flow values significantly affected by the next control, $T_{s,t+1}$. A reference mass flow, $q^{\text{ref}}_{t+j|t}$, can then be obtained as the deterministic equivalent for this stochastic constraint, assuming that the error of the heat load predictions (and the return temperature predictions) are normally distributed, see refs. [2, 7].

2.2.2. *A simple controller.* By observing the return temperature of the water and assuming maximum water circulation, the lowest admissible supply temperature in the next hour is computed from the energy balance equation:

$$p_t = c_w q_t (T_{s,t} - T_{r,t}). \tag{4}$$

The simplest possible controller is then:

$$T_{s,t+1} = \hat{T}_{r,t+1|t} + \frac{\hat{p}_{t+1|t}}{c_w q^{\text{ref}}_{t+1|t}}. \tag{5}$$

2.2.3. *An advanced flow sub-controller—weighted predictive control.* It is obvious that a temporary change of the supply temperature affects the consumers after different time-delays. Therefore it is relevant to take heat load predictions with different horizons into consideration when controlling the supply temperature (the mass flow). Hence, a multi-step predictive controller is required. Such controllers (weighted predictive controller) are proposed in refs. [2, 7].

The main idea of the weighted-predictive controller is as follows: for each prediction horizon, j, ($j \in [N_1, \ldots, N_2]$, see equation (3)) the desired supply temperature, for that particular horizon, is found as:

$$T^{(j)}_{s,t+1} = \hat{T}_{r,t+j|t} + \frac{\hat{p}_{t+j|t}}{c_w q^{\text{ref}}_{t+j|t}}. \tag{6}$$

A reasonable j-step predictor for the return temperature is $\hat{T}_{r,t+j|t} = T_{r,t}$, due to minor variations. The desired supply temperature, for all the prediction horizons, is then constructed as a weighted average:

$$T_{s,t+1} = \sum_{j=N_1}^{N_2} \gamma_j T^{(j)}_{s,t+1}, \quad \left(\sum_{j=N_1}^{N_2} \gamma_j = 1 \right), \tag{7}$$

where each of the weights, γ_j, is chosen as the fraction of the heat consumed at the particular time-delay j.

3. THE SUPPLY TEMPERATURE SUB-CONTROLLER

3.1. *The transfer function model*

In order to find an optimal control, a model of the district heating network is needed. If a perfect description of the entire network is available, a pure physical model can be obtained. Unfortunately, for large networks this type of model can be almost impossible to establish and, if a model can be established, it is likely to be too complex for control purposes. Therefore, in this work statistical transfer functions are identified. Each transfer function describes the relation between the supply temperature from the district heating plant and the supply temperature at a representative point in the network.

The transfer function model considered belongs to the single-input single-output ARX (Auto-Regressive-eXtraneous) structure, see ref. [12] for more details. In ref. [13] it is shown how these models are identified and an example of a model for the district heating system in Esbjerg is:

$$T_{n,t} = a_1 T_{n,t-1} + b_{2,t} T_{s,t-2} + b_{3,t} T_{s,t-3} + b_{4,t} T_{s,t-4} + \epsilon_t. \tag{8}$$

In this model example the time-delay is equal to two time steps (hours). The time-delay in the system actually shows considerable variation due to the variation of the mass flow. The values of the time-delay and the parameters are estimated from hourly measurements of supply temperatures ($T_{s,t}$ and $T_{n,t}$) and the estimates are updated once each hour (see refs. [6, 13] for details).

3.1.1. *The time-variation (non-stationarity).* A district heating system is a non-stationary system and, therefore, time-varying parameters have to be introduced. The time-variations of the parameters are mainly of two kinds: slow annual variations, which are due to changes of the dynamic characteristics of the distribution network (induced by seasonal climate changes), and faster diurnal variations induced by varying heat consumption. The slow variations can be coped with by using adaptive estimation techniques, while the diurnal variation is included as an explicit part of the model.

3.1.2. *The time-delay.* The time-delay in district heating systems can be relatively large compared to the sampling interval (in Esbjerg the time-delays are from 2 to 10 h). Since the time-delay is time-varying and a function of several variables, like distance, flow in the pipes and intersection lay-out of the network, the identification of the delay is one of the major problems in modelling district heating systems. Statistical methods for estimating or tracking such a time-varying time-delay are developed and described in detail in refs. [6, 14]. Hence, this problem will not be addressed in this paper.

3.1.3. *The predictions.* The optimal predictions are found using conditional expectations, see for example, refs. [12] or [15], conditioned on the information available at the present time. The predictions are decomposed into two parts, one depending on information known at the present time t (the past controls $(T_{s,t-1}, T_{s,t-2}, \ldots)$ and the present and past outputs $(T_{n,t}, T_{n,t-1}, \ldots)$), and another depending on information not available (known) at the present time t (the present and the future controls $(T_{s,t}, T_{s,t+1}, \ldots)$). Furthermore, the controllers utilize the fact that the predictions and the prediction errors are uncorrelated, whereby the problem to be minimized can be decomposed into a deterministic part and a stochastic part.

3.2. *The reference temperature*

Typically, the temperature requirements are given by a minimum supply temperature curve, which expresses the supply temperature as a function of the ambient air temperature (Fig. 2). The supply temperature curve may be a mutual agreement between the district heating company and

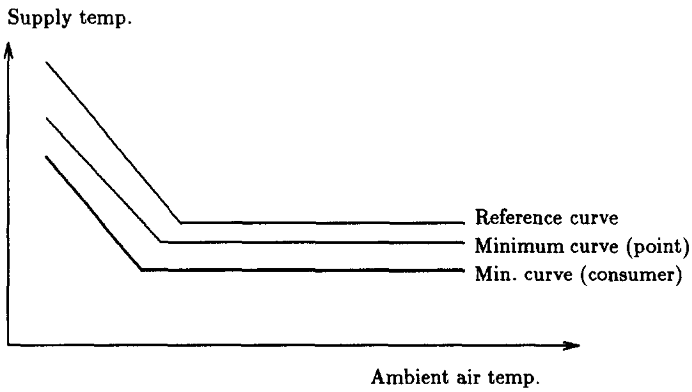

Fig. 2. The minimum temperature curve at the consumers, at the representative point, and the reference temperature curve.

the consumers. Each representative point has its own minimum curve, depending on its location in the network. The minimum curves used are all of the same form as shown in Fig. 2. The minimum temperature is constant for ambient air temperatures higher than a certain value and for lower ambient air temperature it increases linearly with falling ambient air temperature.

The reference value is determined such that the supply temperature is above the minimum supply temperature curve, for any particular point with some prespecified probability [2].

Since the guaranteed minimum temperature depends on the ambient air temperature and because of the time-delay in the system, it is necessary to have predictions of the future ambient air temperature. The computer program PRESS uses models for ambient air temperature described in ref. [6]. The models are based on exponential smoothing. In the calculation of the reference temperature both the uncertainty of the prediction of the supply temperature at the representative points and the uncertainty of the prediction of the ambient air temperature are taken into account.

3.3. *The sub-controller*

3.3.1. *A simple controller.* In the present case the aim is to minimize the supply temperature from the district heating plant by observing that the supply temperatures in the network (at the representative points) should be kept above the minimum curve at least 99% of the time. Therefore, it is desirable to keep the reference supply temperature (set point) as low as possible; this can be done by minimizing the variance of the supply temperature from the plant (Fig. 3), leading to a minimum variance (MV) control.

3.3.1.1. *Minimum variance control.* A simple description of the MV controller is as follows: assume that the time-delay (the transport-delay of the water) is k time steps. The k-step prediction of the supply temperature is then simply set equal to the reference supply temperature for the same time step:

$$\hat{T}_{n,t+k|t} = T^{\text{ref}}_{n,t+k|t}. \tag{9}$$

The supply temperature from the plant (the control variable), $T_{s,t}$, can then easily be solved from equation (9), as $\hat{T}_{n,t+k|t}$ is a function of $T_{s,t}$, cf. equation (8).

3.3.2. *Extended controllers.* In spite of its simplicity the MV controller has some disadvantages: It highly depends on the fact that the underlying process is accurately represented by the identified model, i.e. the order of the model and the time-delay are accurate, and instability may be encountered if the model is non-minimum phase. Furthermore, as the controller is solely concerned about the output, $T_{n,t}$, and not about the control signal, $T_{s,t}$, this may result in an undesirable variation of the control signal. It is obvious that changing the supply temperature rapidly is not desirable as it implies wearing of the pipelines and causes undesirable operation of the plant.

In order to circumvent this problem some extensions to the controller are proposed.

3.3.2.1. *Linear quadratic Gaussian control (LQG).* This type of controller is presented and analysed in ref. [16]. By using this controller it is possible to penalize variations in the supply temperature from the plant by means of a penalty parameter It is shown in ref. [16] how various values of the penalty parameter influence the control performance and the control effort. The main result of the simulations in ref. [16] is that, for small positive values of the penalty, the variation in the supply temperature from the plant is reduced considerably without aggravating the control performance (the variance of the supply temperature at the particular point in the network).

This controller eliminates some of the drawbacks of the MV controller, but it is still sensitive to inaccurate time-delay and model order.

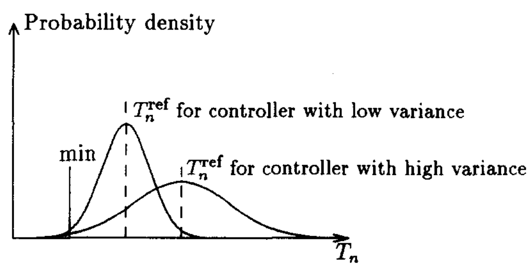

Fig. 3. Expressing regulation performance in terms of variation in output (error) variables.

3.3.2.2. *Generalized predictive control for non-stationary systems.* Due to the thermal capacity of the district heating system the changes in the supply temperature, $T_{s,t}$, do not only influence the supply temperature at the representative point after k time steps, $T_{n,t+k}$, (k is still the time-delay), but also at subsequent time-delays, $T_{n,t+k+1}$, $T_{n,t+k+2}, \ldots$. Therefore, it is natural to extend the controller in order to take care of this, i.e. the controller endeavours not only to keep $T_{n,t+k}$ close to the reference $T^{ref}_{n,t+k|t}$ (see MV and LQG), but also to keep $T_{n,t+j}$ close to $T^{ref}_{n,t+j|t}$ for all relevant time-delays j, $j \in [N_1, \ldots, N_2]$.

Furthermore, it is not only desirable to minimize the change in the present control signal, $T_{s,t}$, as in the previously described LQG controller, but also to keep the variations of the subsequent controls, $\Delta T_{s,t}, \ldots, \Delta T_{s,t+N_u-1}$, where N_u is the control horizon, down.

These extensions, in fact, lead to a controller called the Generalized Predictive Controller (GPC), which was first presented, for time-invariant systems, in ref. [17]. In order to circumvent the problem, concerning the time-varying system, some modifications to the classical GPC are proposed in ref. [18]. Furthermore, the underlying model, in the present case, is an ARMAX model instead of the ARIMAX model used in the original version of the GPC, ref. [17], and almost all later references; this implies some further modifications [18].

The proposed, modified GPC controller is studied in ref. [19] in the district heating context by simulation. The conclusions in ref. [19] are that the effect of penalizing the changes in the supply temperature is similar to that for the previously described LQG controller. The effect of various control horizons, N_u, is also studied. The "best" controller is obtained with $N_u = 2$, 3, or 4 and for small positive values of the penalty parameter.

Furthermore, it has been shown that the Generalized Predictive Controller is quite robust, i.e. non-sensitive to inaccurately estimated time-delays and incorrect model order.

4. THE OVERALL CONTROLLER

As previously mentioned, the overall controller selects the highest supply temperature from among the required supply temperatures calculated from the flow sub-controller and from the supply temperature sub-controllers. This temperature is used as the supply temperature for the plant in the following hour; the model parameters, the predictions and the controller signals are updated each hour (Fig. 1).

5. PRACTICAL EXPERIENCE

The computer program PRESS has been in use for several years at Vestkraft, in Esbjerg, where five representative points in the network have been used, with excellent results. In order to assess the savings the following rules of thumb have been used:

(i) the heat energy price at the CHP plant is assumed to decrease with falling plant supply temperature (about 0.7%°C^{-1})
(ii) the heat loss from the network is assumed to decrease with falling plant supply temperature (about 0.5%°C^{-1})

The savings are analysed in [2]. It is concluded that in the ambient air temperature interval, where data is available, the use of PRESS has implied a lowering of the supply temperature of about 9°C, compared with the control used before PRESS was brought into operation. Consequently, the fuel consumption has been lowered considerably in this ambient air temperature interval and hereby implies large savings in production costs.

6. SUMMARY AND CONCLUSIONS

This paper describes a concept to minimize the supply temperature from a CHP plant, where the pumping costs are considered to be of minor importance compared to the heat loss; this is done by several sub-controllers. A flow sub-controller that utilizes the predictions of the future heat load to keep the mass flow near, but with given probability below, the maximum flow limit, keeps the supply temperature from the plant as low as possible. A number of supply temperature sub-controllers, endeavour to keep the supply temperatures at several representative locations in the network close to, but with given probability above, a given ambient air temperature-dependent minimum. Hence, the supply temperature sub-controllers keep the supply temperature from the plant as low as possible and, with some extensions, penalize large variation in the plant supply temperature. The highest supply temperature among the optimal supply temperatures from the sub-controllers is then used as the supply temperature from the plant in the following hour.

The conclusion that can be drawn is that stochastic modelling and predictive control techniques are extremely well suited for controlling supply temperature and mass flow in district heating systems; as they take care of the uncertainties that are embedded in every system in an appropriate way and do not depend on a huge amount of physical data, just on relevant measurement data. Experience has shown that use of these methods has resulted in a considerably lower supply temperature, at least in the major part of the year, and hence considerably lower fuel consumption.

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