ELSEVIER

Energy and Buildings 22 (1995) 67–79

ENERGY AND BUILDINGS

Estimation of continuous-time models for the heat dynamics of a building

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Received 25 August 1994; accepted in revised form 2 September 1994

Abstract

This paper describes a method for estimation of continuous-time models for the heat dynamics of buildings based on discrete-time building performance data. The parameters in the continuous-time model are estimated by the maximum likelihood method where a Kalman filter is used in calculating the likelihood function. The modeling procedure is illustrated by using measurements from an experiment where the heat input from electrical heaters is controlled by a pseudorandom binary signal. For the considered building a rather simple model containing two time constants is found adequate. Owing to the continuous-time formulation the parameters of the model are directly physically interpretable. The performance of the model for both forecasting and simulation is illustrated.

Keywords: Heat dynamics; Continuous-time models; Residential buildings

1. Introduction

In order to derive a total model for the heat dynamics of a building two different approaches can be used. The traditional approach is simply to use the knowledge of physical characteristics and well-established models of subprocesses. An alternative approach is to use building performance data and statistical methods.

A serious drawback of the traditional approach is the difficulties involved with obtaining a reasonable parameterization. Generally the total model ends with a rather large number of parameters, and, owing to the unavoidable idealizations and simplifications, introduced both into the models of each of the individual subprocesses and into the coupling between the various subprocesses, it is very difficult to predict the accuracy of the total model. This problem is not present with the alternative approach since it is possible, by statistical methods, to deduce whether a given parameterization is reasonable, and, if not, how it can be improved.

A special problem of the traditional approach is to achieve a reasonable description of the short-time dynamical properties. Such a description is essential if the short-time variations of the room air temperature are important, as for instance when controllers containing a feedback from the room air temperature are considered. By using the alternative, statistical approach it is possible to describe variations on the whole time scale covered by the experimental data, and thus also the short-time dynamical properties provided the sampling time is chosen properly.

In the last decades the alternative approach to obtaining equations describing the heat dynamics of buildings has been used still more frequently. Kusuda et al. [1] have used experimental data and ordinary least squares estimation to obtain a first-order difference equation for the dynamics of the thermal mass of a building, and Letherman et al. [2] have used experimental data based on pseudorandom binary sequences of the input to determine the heat dynamics. A rather flexible type of model is treated by Troelsgård [3], who has used ARMAX processes when discussing dynamical models of the variations of the room air temperature in an occupied office building. Furthermore, Troelsgård discusses in Ref. [3] the importance of the various internal and environmental parameters for the variations of the room air temperature on a short time scale. ARMAX models are also discussed by Crawford and Woods [4], who consider a single-family residence with electric heating. A review on methods for estimating heat dynamic models of buildings in both the time and frequency domain is found in Ref. [5].

0378-7788/95/$09.50
SSDI 0378-7788(94)00904-X

Since the building performance data are sampled at discrete times, the model is most frequently formulated in discrete time as a difference equation. However, one serious drawback of the discrete-time formulation is that information about the physical parameters is partially hidden in the discrete-time parameterization. Furthermore it is frequently impossible, based on a discrete-time formulation, to find a reasonable continuous-time model, owing to observational errors or limitation in the flexibility of the model. When it is impossible to obtain a suitable continuous-time formulation it is also impossible to change the sampling time properly. Hence it is desirable to use an estimation method, where the parameterization is kept in continuous time. Furthermore, this procedure ensures a more reasonable physical interpretation of the parameters, and it allows us to use the knowledge of, for example, physical constants or balance relations to improve the parameterization. Finally, if the estimation takes place in continuous time, information about the uncertainty due to quantization of physical characteristics may appear directly as a part of the estimation procedure. Models in continuous time have previously been proposed in Refs. [6–8]. However, these papers only consider deterministic models.

The main purpose of the present paper is to discuss the estimation of a simple continuous-time stochastic model for an experimental building, which explicitly describes how the measurement and model errors enter into the model. Due to the continuous-time formulation it is possible to give a direct physical interpretation of the estimated parameters. It is believed that the approach may contribute to a reduction of the gap between the conventional models based on physical characteristics and the pure empirical discrete-time approach. The relation between the stochastic continuous-time model and ARMAX models is outlined. As a secondary objective the paper discusses statistical methods for deducing whether any given model contains a reasonable description of the observed heat dynamics. Based on these methods it is demonstrated that the very simple model suggested gives a reasonable description of both the observed short- and long-time dynamical behavior of the considered experiment building.

The method described in the paper is selected as the so-called advanced test method in the PASSYS project, which is a common European project related to passive solar energy research [9].

2. The test building and the experiment

The data are from an experiment carried out by the Thermal Insulation Laboratory at the Technical University of Denmark in the period during October 10–14, 1983. The goal of the experiment was to form the basis for an investigation of statistical methods for identification of building equivalent parameters. The data were previously analyzed in Refs. [10,11].

The test building is a single-story wood-built house with crawl and roof space. The floor and the walls are lightweight sandwich constructions based on a masonite beam insulated with 300 mm of mineral wool. The ground floor, which is the test space, is divided into an east and a west room, each of 60 m^2, by a partition wall insulated with 95 mm of mineral wool. The window area makes up 15% of the floor area, 10% facing south and 5% facing north. All the windows are triple glazed. The building is extremely tight. The air change rate has been measured to 0.005 ach by means of a tracer–gas decay method. Electrical heaters are used in both rooms. This form of energy supply makes it possible to consider rather flexible input signals.

In the considered experiment, the room facing east is made thermally heavy by adding 92 m^2 of concrete flags with a thickness of 50 mm. The concrete flags are placed on racks, which ensure a very easy heat transfer by convection between the flags and the surrounding air. In the present context, only measurements from the east room are considered.

Electrical heaters of 3×500 W are used for the energy supply in both rooms. For controlling the supply an on–off control has been used. The actual supply – either on or off – is determined by a pseudorandom binary sequence [12] (PRBS signal) with time period $T=1$ h and order 6, as indicated in Fig. 1. The signal is a deterministic signal with white-noise properties, and shows no correlation with other external signals. Since the signal is deterministic, it can be selected in accordance with the interesting part of the frequency scale of variation.

For the actual experiment the sampling time is 10 min and the following variables are measured:

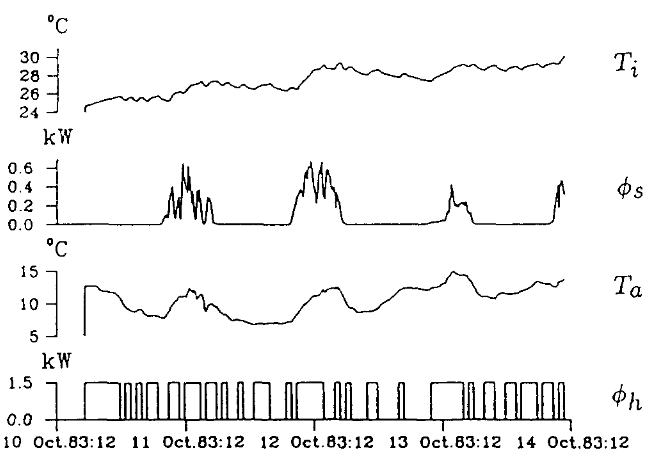


Fig. 1. The measurements of the indoor air temperature, T_i, the solar radiation, ϕ_s, the ambient air temperature, T_a, and the PRBS controlled input from the electrical heaters, ϕ_h. PRBS signal: $N=6$, $T=1$ h.

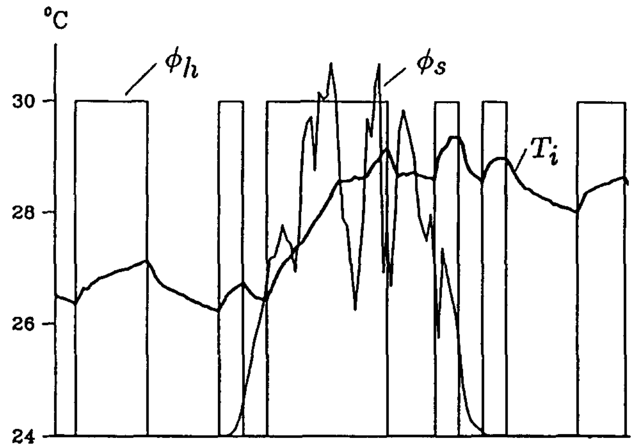


Fig. 2. The measurements of incident solar radiation, input from the electrical heaters and room air temperature during a single day (October 12, 1983).

- T_i room air temperature (°C);
- T_o surface temperature (°C);
- T_a ambient (outdoor) dry-bulb temperature (°C);
- ϕ_h energy input from the electrical heaters (W);
- ϕ_s solar radiation on a vertical surface facing south (W/m^2).

Furthermore, some measurements from the first floor of the test building, which is used as an ordinary office, are recorded.

The most interesting measurements are shown in Fig. 1; 534 observations of each variable are collected. In order to illustrate the covariation of the room air temperature, the solar radiation and the input from the electrical heaters, measurements of these variables are depicted together in Fig. 2 during a single day. The Figure shows that the room air temperature is profoundly affected by variations of both the solar radiation and the input from the electrical heaters. Considering the variation of the room air temperature, both a rather quick response just after the input is shifted and a more persistent response are recognized. This indicates that at least a second-order dynamical model is required for describing the variations of the room air temperature.

The experiments are more deeply discussed in Ref. [13], and the test building in Ref. [14].

3. Formulation of a model

In this Section the formulation of a simple model for the variations of the room air temperature is described. The model is set up in continuous time and the so-called equivalent thermal parameters (ETPs) are introduced. The final model is illustrated by electrical symbols in accordance with the commonly used thermal–electrical analogy.

In the traditional approach the formulation of the model is based on knowledge of physical characteristics for the materials used and of submodels for heat conduction, convection and radiation. The total model of the heat transfer is thus (mostly) based on a simplified diffusion equation for the heat conduction in the walls, an equation for convectional heat transfer between the air and the surfaces, and some equations for the radiative transfer between surfaces. In a commonly used simplification, which is originally found in Ref. [15], the entire heat capacity of the materials is concentrated in a single heat-storing medium. Frequently this medium is thought of as a thin layer in the middle of the walls. If we define the following vector containing the external variables

$$U=(T_a, \phi_h, \phi_s)' \tag{1}$$

then a very simple total model for the indoor temperature T can be written as

$$c\frac{dT}{dt}=-\frac{T}{r}+\boldsymbol{b}U \tag{2}$$

where c is the entire heat capacity of the building, and r is a constant. The constant vector $\boldsymbol{b}$ determines how the external influence, U, enters the system. The model, Eq. (2), contains only a single time constant which is equal to rc. Since the single heat capacity in the model is supposed to describe the entire heat capacity of the building, the model is able only to describe the long-time variations.

Without influence from the sun and the electrical heaters, it is natural to assume that at stationary conditions in Eq. (2), the indoor temperature T is equal to the ambient air temperature. This implies that the first element in the vector $\boldsymbol{b}$ is equal to $1/r$. In this case r can be regarded as the resistance against heat flux between the indoor air and the ambient air.

The very simple model, Eq. (2), is frequently considered in the literature, for example, in Refs. [5,16,17]. Others have used a slightly modified version of Eq. (2), for instance in Ref. [10] it is found that the simple model was unable to describe the short-time dynamics. In order to overcome this problem they have modeled the short-time dynamics as a step change just after a shift in the heat input, but they suggest that an improved model must consider two time constants.

Also in the present investigation the observed variation of the room air temperature suggests at least two time constants – cf. Fig. 2. The heat capacity is dominated by the concrete flags placed on the floor of the building in the middle of the room. Since the outer wall of the considered test building is a very lightweight construction, it may then be reasonable to consider its heat capacity to be negligible. The heat supply from the sun and the radiators either reaches the indoor air by convection or the surfaces by radiation.

The above-mentioned facts and assumptions lead to the model suggested in Fig. 3. The states of the model are given by the temperature T_m of the large heat-accumulating medium with the heat capacity c_m, and by the temperature T_i of the room air and possibly the inner part of the walls with the capacity c_i. r_i is the resistance against heat transfer between the room air and the large heat-accumulating medium, while r_a is the resistance against heat transfer from the room air to the ambient air with the temperature T_a.

The input energy is supplied by the electrical heaters, ϕ_h, and the solar radiation which penetrates through the windows facing south, $A_w \phi_s$, where A_w is the effective window area. The effective window area is the window area corrected for shade effects, absorption and reflection by the triple-glazed windows.

Based on Fig. 3 and the comments above about the energy supplies, the following model is proposed for the heat dynamics of the building:

$$\begin{bmatrix} \frac{dT_m}{dt} \\ \frac{dT_i}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-1}{r_i c_m} & \frac{1}{r_i c_m} \\ \frac{1}{r_i c_i} & -\left(\frac{1}{r_a c_i} + \frac{1}{r_i c_i}\right) \end{bmatrix} \begin{bmatrix} T_m \\ T_i \end{bmatrix} + \begin{bmatrix} 0 & 0 & \frac{A_w p}{c_m} \\ \frac{1}{r_a c_i} & \frac{1}{c_i} & \frac{A_w(1-p)}{c_i} \end{bmatrix} \begin{bmatrix} T_a \\ \phi_h \\ \phi_s \end{bmatrix} \tag{3}$$

The temperature, T_i, of the room air (and the inner part of the walls) and the temperature, T_m, of the large heat-accumulating medium are the two states in the model. The constants c_m, c_i, r_a, r_i, A_w and p are equivalent thermal parameters, which describe the dynamical behavior of the building. p is the part of the solar radiation which is directly affecting T_m.

Considering buildings where the heat capacity of the outer wall is negligible compared to the heat capacities inside the building, the above model describes an obvious extension to a model containing the two time constants of the simple model given in Eq. (2). If it is necessary to consider the heat capacity of the outer wall separately, it is possible to reformulate the model.

Since the model proposed in Eq. (3) contains two time constants, it is theoretically able to describe both the long- and the short-term dynamics. In Section 5 (Results and discussion), statistical methods are used to illustrate whether the model in Eq. (3) also in practice can describe the observed long- and short-time variation.

In matrix notation the equations can be concatenated in *the deterministic linear state space model in continuous time*:

$$\frac{d\boldsymbol{T}}{dt} = \mathbf{A}\boldsymbol{T} + \mathbf{B}\boldsymbol{U} \tag{4}$$

where $\boldsymbol{T}$ is the state vector and $\boldsymbol{U}$ is the input vector. The dynamical behavior of the system is characterized by the matrix $\mathbf{A}$, and $\mathbf{B}$ is a matrix which specifies how the input signals (outdoor air temperature, solar radiation, heat supply, etc.) enter the system.

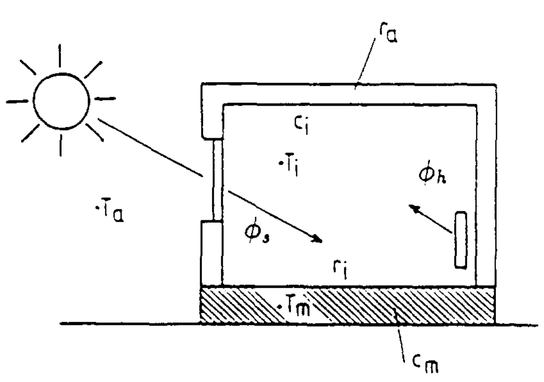
Most frequently, however, Eq. (4) is not able to describe the deviation between Eq. (4) and the true variation of the states, an additive noise term is introduced. Then the model of the heat dynamics is described by the stochastic differential equation

$$d\boldsymbol{T} = \mathbf{A}\boldsymbol{T}\, dt + \mathbf{B}\boldsymbol{U}\, dt + d\boldsymbol{w}(t) \tag{5}$$

where the stochastic process $\boldsymbol{w}(t)$ is assumed to be a process with independent increments. Eq. (5) is *the stochastic linear state space model in continuous time*.

There are many reasons for introducing such a noise term:

- modeling approximations – for instance the dynamics, as described by the matrix $\mathbf{A}$ in Eq. (5), might be an approximation to the true system;



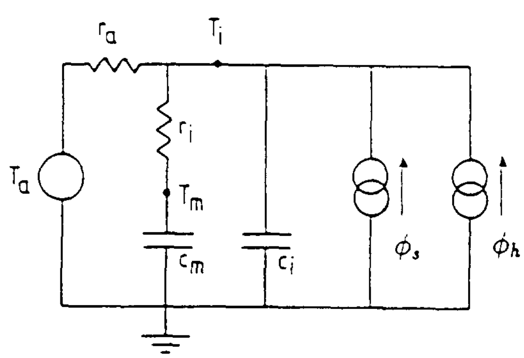


Fig. 3. A two-time-constant model of the heat dynamics of a building.

- unrecognized and unmodeled inputs – some variables which are not considered, e.g., wind speed, may affect the system;
- measurements of the input are noise-corrupted – in this case the measured input is regarded as the actual input to the system, and the deviation from the true input contributes to $w(t)$.

Eq. (5) describes the evolution of all states in the system, but it is most likely that only some of the states are measured. If, for instance, we consider the state space model, Eq. (3), it is reasonable to assume that the temperature of the indoor air is measured, but not the temperature of the large heat-accumulating medium (it might also be difficult to find a reasonable temperature to measure in order to represent the temperature of the large heat-accumulating medium). In the general case we assume that only a linear combination of the states is measured, and if we introduce T_r to denote the measured or recorded variables we can write

$$T_r(t) = \mathbf{C}T(t) + \mathbf{D}U(t) + e(t) \tag{6}$$

where $\mathbf{C}$ is a constant matrix which specifies the linear combinations of the states that actually are measured, and $\mathbf{D}$ is a constant matrix which accounts for input variables which directly affect the output. The equation is, for obvious reasons, called the measurement equation.

The term $e(t)$ is the measurement error. The sensors that measure the output signals are affected by noise and drift. In the following it is assumed that $w(t)$ and $e(t)$ are mutually uncorrelated, which seems to be quite reasonable.

For the considered experiment, where the system is described by Eq. (3), and only the indoor air temperature is measured, the measurement equation simply becomes

$$T_r(t) = [0 \;\; 1] \begin{bmatrix} T_m(t) \\ T_i(t) \end{bmatrix} + e(t) \tag{7}$$

where $e(t)$ is the measurement error which accompanies the measurement of the indoor air temperature.

4. A maximum likelihood method for parameter estimation

In this Section it is shown how the parameters in a linear stochastic differential equation, such as the previously formulated model for the heat transfer, can be estimated by using discrete-time measurements and the maximum likelihood method. This Section may be omitted without problems in understanding the subsequent Sections.

It is assumed that the model of the heat dynamics is described by the stochastic differential equation, Eq. (5), where the state vector T is m-dimensional. With the purpose of calculating the likelihood function, the m-dimensional stochastic process $w(t)$ is restricted to be a Wiener process with the incremental covariance $\mathbf{R}_1^c(t)\,dt$.

The measured variables are given by Eq. (6). It is assumed that the measurement error $e(t)$ is normally distributed white noise with zero mean and variance $\mathbf{R}_2$. Furthermore, it is assumed that $w(t)$ and $e(t)$ are mutually independent.

It is a crucial question whether the parameters of a specified state space model can be identified. If a nonidentifiable model is specified, the methods for estimation will not converge. The problem of identifiability arises from the fact that for a given transfer function model, there corresponds, in general, a whole continuum of possible state space models. Therefore, we must introduce a restriction on the structure of the state space model, in order to provide a unique relation between the unknown parameters of the state space model and those of the transfer function. It can be shown that the model given in Eq. (3) can be identified – see Ref. [13].

4.1. From continuous to discrete time

Since it is assumed that the system is described by the stochastic differential equation, Eq. (5), it is possible analytically to perform an integration which, under some assumptions, exactly specifies the system evolution between discrete time instants.

The discrete-time model corresponding to the continuous-time model in Eq. (5) is obtained by integrating the differential equation through the sample interval $[t, t+\tau)$. Thus the sampled version of Eq. (5) can be written as

$$\begin{aligned} T(t+\tau) = {} & \exp[\mathbf{A}(t+\tau-t)]T(t) \\ & + \int_t^{t+\tau} \exp[\mathbf{A}(t+\tau-s)]\mathbf{B}U(s)\,ds \\ & + \int_t^{t+\tau} \exp[\mathbf{A}(t+\tau-s)]\,dw(s) \end{aligned} \tag{8}$$

Under the assumption that $U(t)$ is constant in the sample interval, the sampled version can be written as the following discrete-time model in state space form:

$$T(t+\tau) = \Phi(\tau)T(t) + \Gamma(\tau)U(t) + v(t;\tau) \tag{9}$$

where

$$\Phi(\tau) = \exp(\mathbf{A}\tau); \qquad \Gamma(\tau) = \int_0^{\tau} \exp(\mathbf{A}s)\mathbf{B}\,ds \tag{10}$$

$$v(t;\tau) = \int_t^{t+\tau} \exp[\mathbf{A}(t+\tau-s)]\,dw(s) \tag{11}$$

Using the assumption that $w(t)$ is a Wiener process, $v(t;\ \tau)$ becomes normally distributed white noise with zero mean and covariance

$$\mathbf{R}_1(\tau) = E[v(t;\ \tau)v(t;\ \tau)'] = \int_0^{\tau} \Phi(s)\mathbf{R}_1^c\Phi(s)'\ \mathrm{d}s \tag{12}$$

If the sampling time is constant (equally spaced observations), the stochastic difference equation can be written

$$T(t+1) = \Phi T(t) + \Gamma U(t) + v(t) \tag{13}$$

where the time scale now is transformed such that the sampling time becomes equal to one time unit.

4.2. Maximum likelihood estimates

In the following it is assumed that the observations are obtained at regularly spaced time intervals, and hence that the time index t belongs to the set $\{0, 1, 2, \ldots, N\}$. N is the number of observations. In order to obtain the likelihood function we further introduce

$$\mathbf{T}_r(t) = [T_r(t), T_r(t-1), \ldots, T_r(1), T_r(0)]' \tag{14}$$

i.e., $\mathbf{T}_r(t)$ is a matrix containing all the observations up to and including time t. Finally, let $\boldsymbol{\theta}$ denote a vector of all the unknown parameters – including the unknown variance and covariance parameters in $\mathbf{R}_1$ and $\mathbf{R}_2$.

The likelihood function is the joint probability density of all the observations assuming that the parameters are known, i.e.,

$$\begin{aligned} L'(\boldsymbol{\theta};\ \mathbf{T}_r(N)) &= p(\mathbf{T}_r(N)|\boldsymbol{\theta}) \\ &= p(T_r(N)|\mathbf{T}_r(N-1),\ \boldsymbol{\theta})p(\mathbf{T}_r(N-1)|\boldsymbol{\theta}) \\ &= \left(\prod_{t=1}^{N} p(T_r(t)|\mathbf{T}_r(t-1),\ \boldsymbol{\theta})\right)p(T_r(0)|\boldsymbol{\theta}) \end{aligned} \tag{15}$$

where successive applications of the rule $P(A\cap B) = P(A|B)P(B)$ are used to express the likelihood function as a product of conditional densities.

Since both $v(t)$ and $e(t)$ are normally distributed the conditional density is also normal. The normal distribution is completely characterized by the mean and the variance. Hence, in order to parameterize the conditional distribution, we introduce the conditional mean and the conditional variance as

$$\hat{T}_r(t|t-1) = E[T_r(t)|\mathbf{T}_r(t-1),\ \boldsymbol{\theta}] \tag{16}$$

and

$$\mathbf{R}(t|t-1) = V[T_r(t)|\mathbf{T}_r(t-1),\ \boldsymbol{\theta}] \tag{17}$$

respectively. Note that Eq. (16) is the one-step prediction and Eq. (17) is the associated variance. Furthermore, it is convenient to introduce the one-step prediction error (or innovation):

$$\boldsymbol{\epsilon}(t) = T_r(t) - \hat{T}_r(t|t-1) \tag{18}$$

Using Eqs. (16)–(18) the conditional likelihood function (conditioned on $T_r(0)$) becomes

$$L(\boldsymbol{\theta};\ \mathbf{T}_r(N)) = \prod_{t=1}^{N}\left\{(2\pi)^{-m/2} \det \mathbf{R}(t|t-1)^{-1/2} \times \exp\left[-\frac{1}{2}\boldsymbol{\epsilon}(t)'\mathbf{R}(t|t-1)^{-1}\boldsymbol{\epsilon}(t)\right]\right\} \tag{19}$$

where m is the dimension of the vector T_r. Most frequently the logarithm of the conditional likelihood function is considered. It is written

$$\log L(\boldsymbol{\theta};\ \mathbf{T}_r(N)) = -\frac{1}{2}\sum_{t+1}^{N}[\log \det \mathbf{R}(t|t-1) + \boldsymbol{\epsilon}(t)'\mathbf{R}(t|t-1)^{-1}\boldsymbol{\epsilon}(t)] + \text{const.} \tag{20}$$

The conditional mean $\hat{T}_r(t|t-1)$ and the conditional variance $R(t|t-1)$ can be calculated recursively by using a Kalman filter – see Refs. [18] or [19]. The Kalman filter is most easy to understand as formulas for recursively calculating a one-step prediction (or estimate) of the state of the system, together with formulas for updating (or reconstructing) this estimate. In the present case, where the transfer of the states of the system in discrete time is described by Eq. (13) and the observations by Eq. (6), the equations for *updating* the estimate of the state T become:

$$\hat{T}(t|t) = \hat{T}(t|t-1) + \mathbf{L}_t[T_r(t) - \mathbf{C}\hat{T}(t|t-1)] \tag{21}$$

$$P(t|t) = \mathbf{P}(t|t-1) - \mathbf{L}_t\mathbf{R}(t|t-1)\mathbf{L}_t' \tag{22}$$

where

$$\mathbf{L}_t = \mathbf{P}(t|t-1)\mathbf{C}'\mathbf{R}(t|t-1)^{-1} \tag{23}$$

The predictive Kalman gain $\mathbf{K}_t$ is

$$\mathbf{K}_t = \Phi\mathbf{L}_t \tag{24}$$

The formulas for *prediction* become:

$$\hat{T}(t+1|t) = \Phi\hat{T}(t|t) + \Gamma U(t) \tag{25}$$

$$\hat{T}_r(t+1|t) = \mathbf{C}\hat{T}(t+1|t) + \mathbf{D}U(t+1) \tag{26}$$

$$\mathbf{P}(t+1|t) = \Phi\mathbf{P}(t|t)\Phi' + \mathbf{R}_1 \tag{27}$$

$$\mathbf{R}(t+1|t) = \mathbf{C}\mathbf{P}(t+1|t)\mathbf{C}' + \mathbf{R}_2 \tag{28}$$

The filter requires some initial values, which describe the prior knowledge about the states of the system in terms of the prior mean and variance:

$$\hat{T}(1|0) = E[T(1)] = \boldsymbol{\mu}_0 \tag{29}$$

$$\mathbf{P}(1|0) = V[T(1)] = \mathbf{V}_0 \tag{30}$$

The matrix $\mathbf{P}(t+1|t)$ is the variance of the one-step prediction of the state T of the system. In the considered case the value of μ_0 is equal to the first observation (for both state variables), and $V_0 = 10^4$.

The recursive use of the Kalman filter as it is formulated above can be explained in the following way: assume that the time is $(t-1)$, and we have calculated the prediction and the associated variance for the state at time t. When the next observation ($T_r(t)$) at time t becomes available, Eqs. (21)–(23) can be used for updating the estimate of the state. Using the updated values it is then possible to calculate the prediction and the associated variance for the state at time $(t+1)$.

The maximum likelihood estimate (ML estimate) is the set $\hat{\theta}$ which maximizes the conditional likelihood function. The IMSL-routine DBCONF [20] was used in the maximization of the likelihood function.

An estimate of the uncertainty of the parameters is obtained by the fact that the ML estimator is asymptotically normally distributed with mean θ and variance

$$\mathbf{D} = \mathbf{H}^{-1} \quad (31)$$

where the matrix $\mathbf{H}$ is given by

$$\{h_{lk}\} = -E\left[\frac{\partial^2}{\partial\theta_l \partial\theta_k} \log L(\theta; \mathrm{T}_r(N))\right] \quad (32)$$

An estimate of $\mathbf{D}$ is obtained by equating the observed value with its expectation and applying

$$\{h_{lk}\} \approx -\left(\frac{\partial^2}{\partial\theta_l \partial\theta_k} \log L(\theta; \mathrm{T}_r(N))\right)\Bigg|_{\theta=\hat{\theta}} \quad (33)$$

The above equation is thus used for estimating the variance of the parameter estimates. If an estimated variance is large compared to the actual estimated value for a parameter, this indicates that probably this parameter can be eliminated from the model (the parameter is assumed to be equal to zero).

The method can be extended to cases where the noise is not perfectly normally distributed. In that case the method is a prediction error method (PEM) – see Ref. [21].

Table 1
Maximum likelihood estimates for the east room with approximate standard errors of the estimates (SEE) shown in parentheses

Parameter	Estimate	
	p free	$p=0$
$\hat{c}_i$ (SEE)	1.183 (0.090) kWh/°C	1.183 (0.030) kWh/°C
$\hat{c}_m$ (SEE)	4.005 (0.514) kWh/°C	3.987 (0.218) kWh/°C
$\hat{r}_i$ (SEE)	0.4789 (0.0479) °C/kW	0.4788 (0.0154) °C/kW
$\hat{r}_a$ (SEE)	29.25 (13.55) °C/kW	29.38 (9.37) °C/kW
$\hat{A}_w$ (SEE)	2.866 (0.552) m²	2.845 (0.139) m²
$\hat{p}$ (SEE)	0.0101 (0.1317)	–
$\hat{R}_{11,1}$ (SEE)	0.00266 (0.00171) °C²	0.00265 (0.00084) °C²
$\hat{R}_{22,1}$ (SEE)	0.00468 (0.00309) °C²	0.00469 (0.00090) °C²
$\hat{R}_2$ (SEE)	0.00019 (0.00016) °C²	0.00019 (0.00006) °C²

A description of the numerical details can be found in Ref. [22]. The method has formed the basis for a general software system for continuous-time linear system modeling [23].

5. Results and discussion

It was argued previously that the lumped model in Eq. (34) is expected to be reasonable for the considered test building:

$$\begin{bmatrix} dT_m \\ dT_i \end{bmatrix} = \begin{bmatrix} \frac{-1}{r_i c_m} & \frac{1}{r_i c_m} \\ \frac{1}{r_i c_i} & -\left(\frac{1}{r_a c_i} + \frac{1}{r_i c_i}\right) \end{bmatrix} \begin{bmatrix} T_m \\ T_i \end{bmatrix} dt + \begin{bmatrix} 0 & 0 & \frac{A_w p}{c_m} \\ \frac{1}{r_a c_i} & \frac{1}{c_i} & \frac{A_w(1-p)}{c_i} \end{bmatrix} \begin{bmatrix} T_a \\ \phi_h \\ \phi_s \end{bmatrix} dt + \begin{bmatrix} dw_m(t) \\ dw_i(t) \end{bmatrix} \quad (34)$$

$$T_r(t) = \mathbf{C}T(t) + e(t) \quad (35)$$

The constants c_m, c_i, r_a, r_i, A_w and p are the equivalent thermal parameters.

The obtained maximum likelihood estimates of the parameters of the model in Eq. (34) are shown in the first column of Table 1. The corresponding approximate standard errors are shown in parentheses.

It is seen that the parameter p, which describes the fraction of the solar radiation which is directly affecting the large heat-accumulating medium, is close to zero. In order to deduce whether it is reasonable to assume that p really is zero, a likelihood ratio test is performed based on the restriction $p=0$. The test has shown that the hypothesis $p=0$ cannot be rejected on any reasonable level. Therefore, it is concluded that the solar radiation is directly affecting only the room air including the inner part of the walls. The second column in Table 1 shows the results under the restriction $p=0$.

The estimated variance of the one-step prediction error (cf. Eq. (18)) of the recorded indoor air temperature, T_r, is $\hat{\sigma}_\epsilon^2 = (0.0316\ ^\circ\mathrm{C})^2$.

At stationary conditions the one-step prediction variance is constant, i.e.,

$$\mathbf{P}(t+1|t) = \mathbf{P}(t|t-1) = \mathbf{P}_\infty \quad (36)$$

By using Eqs. (22), (27) and (36), an equation for $\mathbf{P}_\infty$ is obtained. This is the stationary Riccatti equation (see Ref. [21]):

$$\mathbf{P}_\infty = \boldsymbol{\Phi}\mathbf{P}_\infty\boldsymbol{\Phi}' + \mathbf{R}_1 - \boldsymbol{\Phi}\mathbf{P}_\infty\mathbf{C}'[\mathbf{C}\mathbf{P}_\infty\mathbf{C}' + \mathbf{R}_2]^{-1}\mathbf{C}\mathbf{P}_\infty\boldsymbol{\Phi}' \tag{37}$$

In the estimated system, the stationary one-step prediction variance for the state $\boldsymbol{T}$ is

$$\hat{\mathbf{P}}_\infty = \begin{bmatrix} 0.00164 & 0.00047 \\ 0.00047 & 0.00080 \end{bmatrix} \tag{38}$$

Thus the variance of the one-step prediction error of the state T_i is $\hat{P}_{\infty,22} = 0.00080 \approx (0.028\ °\mathrm{C})^2$. Following Eq. (28) the stationary variance of the one-step prediction error of the recorded variable is

$$\hat{\sigma}_\epsilon^2 = \overbrace{\hat{P}_{\infty,22}}^{\text{System error}} + \overbrace{\hat{R}_2}^{\text{Measurement error}} \approx (0.0315\ °\mathrm{C})^2 \tag{39}$$

which nicely corresponds to the variance of the observed one-step prediction error. It is also seen that the measurement error is small compared to the 'system' error.

The input matrix

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & \dfrac{A_w p}{c_m} \\ \dfrac{1}{r_a c_i} & \dfrac{1}{c_i} & \dfrac{A_w(1-p)}{c_i} \end{bmatrix} \tag{40}$$

determines how the influences from the outdoor air temperature, the solar radiation, and the electrical heaters affect the temperature of the room air and the temperature of the heat-accumulating medium. The zero values in the $\mathbf{B}$ matrix indicate that the outdoor air temperature and the electrical heaters affect the heat-accumulating medium only through the indoor air temperature – and not directly.

The dynamical characteristics are described by the matrix

$$\mathbf{A} = \begin{bmatrix} \dfrac{-1}{r_i c_m} & \dfrac{1}{r_i c_m} \\ \dfrac{1}{r_i c_i} & -\left(\dfrac{1}{r_a c_i} + \dfrac{1}{r_i c_i}\right) \end{bmatrix} \tag{41}$$

By using the estimates in Table 1 for $p=0$, the eigenvalues and eigenvectors of $\mathbf{A}$ become

$\lambda_1 = -2.3121\ \mathrm{h}^{-1}$ and $\lambda_2 = -0.0065\ \mathrm{h}^{-1}$

$$\boldsymbol{v}_1 = \begin{bmatrix} 0.2811 \\ -0.9597 \end{bmatrix} \text{ and } \boldsymbol{v}_2 = \begin{bmatrix} 0.7115 \\ 0.7026 \end{bmatrix}$$

respectively. Based on the eigenvalues, the time constants are found as

$$\tau_i = -1/\lambda_i; \qquad i = 1, 2 \tag{42}$$

which, given the above eigenvalues, lead to the following time constants:

$\tau_1 = 26$ min and $\tau_2 = 154$ h

Any state $\boldsymbol{T} = (T_m, T_i)'$ can be described as a linear combination of the eigenvectors. By considering the eigenvectors and eigenvalues for the estimated system, it is shown in the next Section that the long-time variations are related to situations where the room air temperature and the temperature of the heat-accumulating medium are almost equal, whereas the short-time variations are roughly related to a difference between the two temperatures.

5.1. Transient trajectories

When the external influence signals are constant, a stationary value of the state exists which is independent of the initial state vector. The above interpretation of the eigenvalues and eigenvectors becomes more evident by considering the transient trajectories describing how the state of the system, $\boldsymbol{T}(t) = [T_m(t), T_i(t)]'$, approaches the stationary state $\boldsymbol{T}(\infty) = [T_m(\infty), T_i(\infty)]'$ for different choices of the initial state $\boldsymbol{T}(t_0) = [T_m(t_0), T_i(t_0)]'$. When the time parameter is eliminated this is called a phase plot.

Let us assume that the input is $\boldsymbol{U} = (T_a, \phi_h, \phi_s)' = 0$ for $t \geq t_0$, then

$$\boldsymbol{T}(t) = \exp[\mathbf{A}(t-t_0)]\boldsymbol{T}(t_0) \tag{43}$$

Using the eigenvalues and eigenvectors, Eq. (43) can be written as

$$\boldsymbol{T}(t) = z_1(t_0)\exp[\lambda_1(t-t_0)]\boldsymbol{v}_1 + z_2(t_0)\exp[\lambda_2(t-t_0)]\boldsymbol{v}_2 \tag{44}$$

where $[z_1(t_0), z_2(t_0)]$ are the coordinates of the initial state in the vector space spanned by the eigenvectors, i.e.,

$$\begin{bmatrix} z_1(t_0) \\ z_2(t_0) \end{bmatrix} = [\boldsymbol{v}_1 \quad \boldsymbol{v}_2]^{-1} \begin{bmatrix} T_m(t_0) \\ T_i(t_0) \end{bmatrix} \tag{45}$$

Fig. 4 shows the transient trajectories for three different choices of the initial state: $\boldsymbol{T}_1 = (10, 20)'$, $\boldsymbol{T}_2 = (20, 20)'$ and $\boldsymbol{T}_3 = (20, 10)'$.

The initial state $\boldsymbol{T}_3$ might be reasonable for describing a situation where the room has been ventilated recently. The ambient air is supposed to be 0 °C. Shortly after ventilation the temperature of the indoor air will rise due to heat from the heat-accumulating medium and simultaneously the temperature of the heat-accumulating medium will decrease slightly. The direction of the increase in the indoor air temperature and decrease in the temperature of the heat-accumulating layer is mainly determined by $\boldsymbol{v}_1$ and the rate of change is

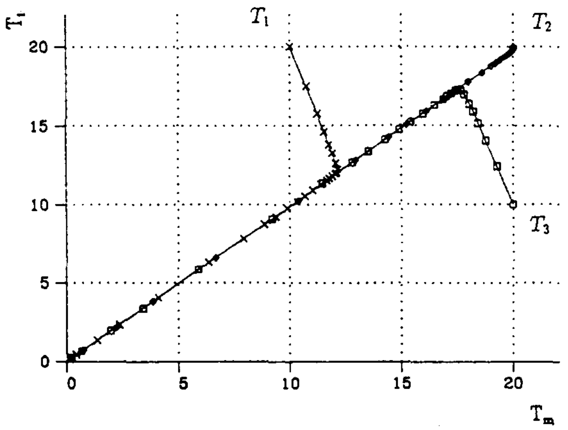


Fig. 4. Transient trajectories for three different initial states.

determined by λ_1. As the temperatures T_1 and T_2 become (nearly) identical a simultaneous decrease of the temperature along the direction $\boldsymbol{v}_2$ will begin (since no external energy source is supposed to be available). The rate of decrease is now determined by λ_2. The above separation in a trajectory, which first is determined by $\boldsymbol{v}_1$ and then by $\boldsymbol{v}_2$, is caused by the fact that the system is extremely stiff, i.e., $\tau_2 \gg \tau_1$. In situations where eigenvalues are of the same order of magnitude, such a separation is not seen as clearly.

5.2. Transfer function representation

If only the relation between the input and output signals of the system is of interest, the transfer function representation might be regarded as a sufficient description of the system. The transfer function is obtained from the stationary state space representation simply by eliminating the states.

Let us restrict our attention to the discrete-time case, and consider the *innovation form* of the discrete-time state space representation:

$$\hat{\boldsymbol{T}}(t+1|t) = \boldsymbol{\Phi}\hat{\boldsymbol{T}}(t|t-1) + \boldsymbol{\Gamma}\boldsymbol{U}(t) + \mathbf{K}\epsilon(t) \quad (46)$$

$$T_r(t) = \mathbf{C}\hat{\boldsymbol{T}}(t|t-1) + \epsilon(t) \quad (47)$$

which is readily obtained by the Kalman filter equations and using the fact that $\mathbf{D}=0$.

By elimination of $\hat{\boldsymbol{T}}(t+1|t)$ in Eqs. (46) and (47), the transfer function representation is obtained, i.e.,

$$T_r(z) = \mathbf{C}(z\mathbf{I}-\boldsymbol{\Phi})^{-1}\boldsymbol{\Gamma}\boldsymbol{U}(z) + \mathbf{C}(z\mathbf{I}-\boldsymbol{\Phi})^{-1}\mathbf{K}\epsilon(z) + \epsilon(z) \quad (48)$$

where z denotes the z-transform variable. Since the order of $\boldsymbol{\Phi}$ is two, the transfer function representation corresponding to the estimated model becomes

$$(z^2+a_1z+a_2)T_r(z) = (b_{1,0}z+b_{1,1})T_a(z) \quad (49)$$

$$+(b_{2,0}z+b_{2,1})\phi_h(z) + (b_{3,0}z+b_{3,1})\phi_s(z)$$

$$+(z^2+c_1z+c_2)\epsilon(z) \quad (50)$$

Based on the estimated parameters for $p=0$ in Table 1, the following transfer function representation is obtained:

$$(z^2-1.6791263z+0.6794737)T_r(z)$$

$$=0.00416(z-0.9166)T_a(z) \quad (51)$$

$$+0.1223(z-0.9166)\phi_h(z)$$

$$+0.3480(z-0.9166)\phi_s(z) \quad (52)$$

$$+(z^2-0.9547z+0.1296)\epsilon(z) \quad (53)$$

The high number of digits stated for the autoregressive part of the transfer function is necessary to obtain reliable steady-state relations, due to the stiffness of the system. Note that the zero of the transfer function is identical for all the input signals, due to the very simple structure of the **B** matrix in the case $p=0$.

The zeros of the autoregressive part, which are the poles of the transfer function from all input signals, become

$$p_1 = -0.6802 \quad (54)$$

$$p_2 = -0.9989 \quad (55)$$

Note that the poles for the discrete-time transfer function might have been calculated from the eigenvalues of the **A** matrix belonging to the continuous-time model by the relationship

$$p_i = \exp(\lambda_i\tau) \qquad i=1, 2 \quad (56)$$

where τ is the sampling time (1/6 h).

5.3. Steady-state equations

If only the steady-state behavior of the system is of interest, one might regard it more convenient to consider a static equation rather than the dynamical equation, Eq. (34). The stationary situation is characterized by $\mathrm{d}T/\mathrm{d}t=0$. Hence it follows that the equation which expresses the static relationship between the influences $\boldsymbol{U}$ and the state $\boldsymbol{T}$ is given by

$$\boldsymbol{T} = -\mathbf{A}^{-1}\mathbf{B}\boldsymbol{U} \quad (57)$$

By using the estimated parameters for $p=0$ in Table 1 we can compute the stationary equations, and a rearrangement of the terms in the equation for T_i yields

$$\phi_h = (0.03404\ \mathrm{kW/^\circ C})(T_i-T_a) - (2.845\ \mathrm{m^2})\phi_s \quad (58)$$

Alternatively the steady-state equation is obtained by putting $z=1$ in the transfer function Eq. (53).

Such a steady-state equation might be used as a simple characterization of the steady-state heat transfer properties of the building. It is important to notice that the above steady-state equation is an outcome of a dynamical model, which then accounts for the serial correlation, which in traditional regression analysis may cause problems.

6. Evaluation of the model

The estimated model is evaluated by three different approaches. Firstly, the estimated parameters are compared with parameters calculated by the traditional approach from the physical characteristics of the building. Secondly, statistical methods are used for verification of the model, and finally, the model performance in simulation and forecast applications is illustrated.

6.1. Comparison with physically evaluated parameters

By the traditional approach a model of the heat dynamics is obtained by using physical constants of the building materials employed, and well-known models of subprocesses which are extrapolated to a model for the whole building. It is, however, very difficult to predict the accuracy of the final model, even when the simple model given in Eq. (2) is considered.

In Ref. [10] values for the parameters in the simple model of Eq. (2) were calculated. The parameters are determined by using physical constants for the building materials employed. For the resistance against heat transfer to the ambient air, r, their calculations give values from 24.7 to 27.7 °C/kW, and for the total heat capacity, c, their calculations give 7.0 kWh/°C. Comparing these with the estimated values in Table 1 and the associated estimation error, it is noticed that the calculated value of r and the estimated value r_a are nearly equal, whereas the calculated value for the total heat capacity is higher than the estimated value.

The transparent window area facing south is 4.9 m^2. Taking the reflection at low angles of incidence into account, around 60% of the incident solar radiation is expected to penetrate through the triple-glazed windows. The effective window area is thus 2.9 m^2, which corresponds very nicely to the estimated value of A_w in Table 1.

As a whole, it is concluded that the estimated parameter values nicely correspond to the calculated values for those parameters which are most easily calculated from basic physical knowledge, but not for the rest of the parameters. Note that some of the estimated parameters have no physically determined counterpart. In a dynamical description of the house, e.g., for simulation purposes, where the heat capacity also becomes important, it is concluded that the simple model of Eq. (2) used with physically determined parameters gives a bad description compared to the estimated extended model.

6.2. Evaluation of the residuals

One point which is often overlooked in discussions about the sufficiency of models for heat dynamics of buildings is that statistical methods can be used to judge whether the model describes all the observed autocorrelation or dynamics of the building. The idea behind the statistical methods is that if the residuals from the estimation of the model can be considered to be a sequence of uncorrelated random elements (white noise), then no more information about the mean value and the correlation properties of the system is left in the residuals and thus all autocorrelation is described by the model.

The estimated autocorrelation function based on all the residuals is shown in Fig. 5. Confidence bands of approximately 95% under the hypothesis that the residuals are white noise are also shown. Since only three of the 24 autocorrelations are just outside the confidence bands, it is reasonable to accept the hypothesis that the residuals belong to an uncorrelated random sequence.

If the residuals contain hidden periodicities, a study of the correlation in the frequency domain can be more useful to reveal such periodicities than the autocorrelation function. The periodogram [24] describes how the variation of the residuals is distributed on frequencies. For a sequence of white noise this variation is equally distributed, i.e., the cumulative periodogram is a straight line from 0 to 1. Fig. 6 shows the cumulative periodogram for the residuals, and confidence bands of approximately 95% under the hypothesis that the residuals are white noise. Since the estimated cumulative periodogram lies inside the confidence bands, it seems reasonable to consider the residuals as a white-noise sequence. Summing up, it is concluded that both the observed time dependence and causality are reasonably well described by the model, which especially means

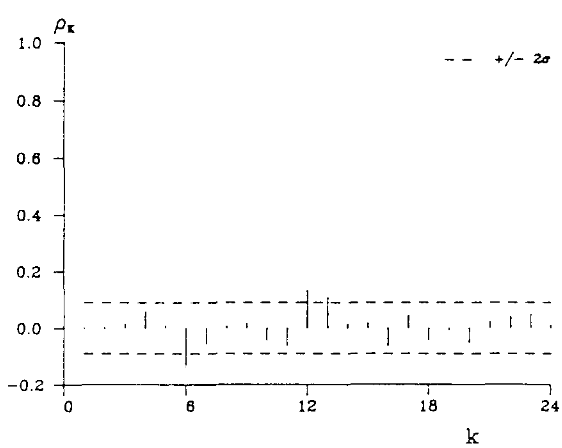


Fig. 5. The estimated autocorrelation of the residuals and corresponding confidence bands of approximately 95%.

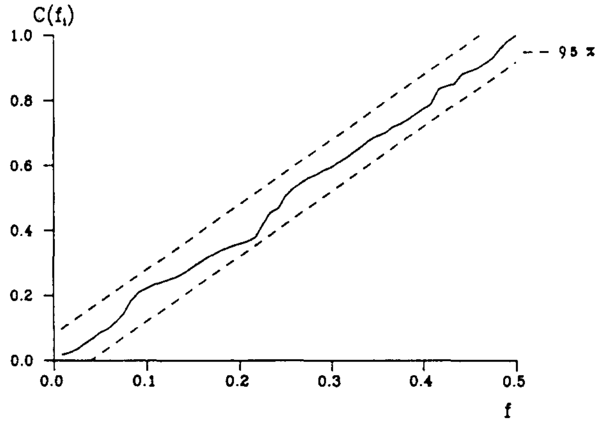


Fig. 6. The cumulative periodogram of the residuals, and corresponding confidence bands of approximately 95%. Linear frequency scale: Kolmogorov–Smirnov approximate probability limits of 95% are shown.

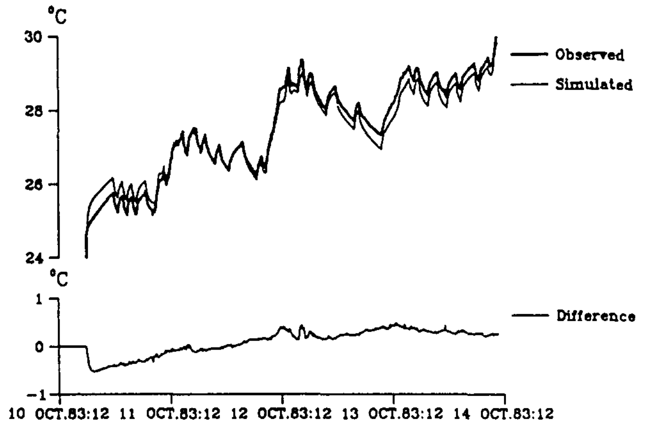


Fig. 7. Simulated and measured room air temperature. Initial value of T_i is 24.62 °C.

that the model order is high enough to describe the system.

6.3. Simulation and forecasting

Generally, the models of the heat dynamics of buildings are to be applied either for simulation or forecasting. This Section illustrates the performance of the formulated model for these two purposes.

Fig. 7 shows the measured and the simulated room air temperature. As a starting point for the simulation $(T_m, T_i) = (25.45\ °C, 24.62\ °C)$ is used. The starting point for the room air temperature is taken as the first observation, whereas the starting value for the heat-accumulating medium is taken as the interception between the estimated straight line and the ordinate, since this temperature is not directly measured. A good agreement between the measured and simulated temperature is observed, and the model describes reasonably well both the long- and the short-time performance of the system.

The performance of the model for forecasting is illustrated in Fig. 8, which shows forecasts 30 min (or 3 steps) ahead of the room air temperature T_i. The one-step forecasts of $T = (T_m, T_i)'$ are calculated as

$$\hat{T}(t+1|t) = \Phi \hat{T}(t|t-1) + \Gamma U(t) + \mathbf{K}\epsilon(t) \tag{59}$$

where $\mathbf{K}$ is the stationary value of $\mathbf{K}(t)$ determined by the stationary conditions of the Kalman filter equations. Hence the n-step forecasts are calculated by repeated use of

$$\hat{T}(t+k+1|t) = \Phi \hat{T}(t+k|t) + \Gamma \hat{U}(t+k|t) \tag{60}$$

for $k = 1, \ldots, n-1$, where $\hat{U}(t+k|t)$ is the predicted influence signal at time $t+k$. In the present example the ambient temperature and the solar radiation at time t are used as the predictions at time $t+k$, whereas the input from the electrical heaters is considered deterministic and therefore totally predictable. This is in accordance with the fact that the input from the electrical heaters is selected before the experiment. The prediction used for the ambient temperature and the solar radiation is thus very simple, and an improved predictability of the room air temperature is naturally obtainable by using more adequate predictions of the influence signals. Models for the outdoor climate which are useful in such applications are described by Madsen [25]. In Ref. [26] it is shown how lumped models for the heat dynamics of buildings can form the basis for an improved control of the heat supply.

In the case of perfect information about the future influences, the variance of the n-step forecast error is calculated by repeated use of

$$\mathbf{W}_{k+1} = \Phi \mathbf{W}_k \Phi' + \mathbf{R}_1 \tag{61}$$

with the starting value $\mathbf{W}_1 = \mathbf{P}(t) = \mathbf{P}$, which is determined from the stationary conditions of the Kalman filter. Based on the estimates in Table 1 the variance of the forecast error for the predictions 30 min ahead becomes $(0.042\ °C)^2$ in the case of perfect information. However, owing to the uncertain predictions of the influences,

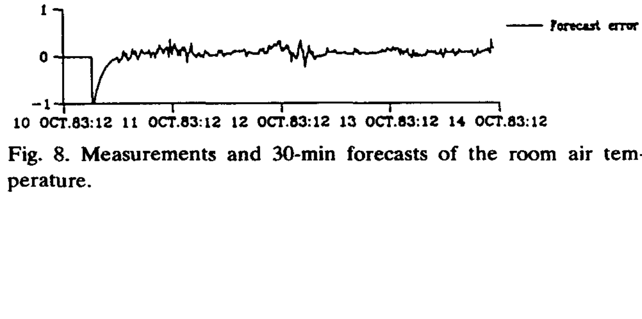


Fig. 8. Measurements and 30-min forecasts of the room air temperature.

the variance of the forecast error for the forecasts shown in Fig. 8 can be estimated to be $(0.058\ ^\circ C)^2$.

7. Conclusions

A procedure for estimation of continuous-time models for the heat dynamics of buildings based on building performance data measured in discrete time is proposed. The fact that the model formulation and modeling is done in continuous time implies that the model can be iteratively improved easily by a combination of more detailed comparisons with data and use of physical facts. By this approach the problems associated with the modeling approaches mainly used today are reduced or eliminated.

The serious problem associated with the traditional approach, which uses the knowledge of physical characteristics to derive a total model for the building, is that it is impossible to predict the accuracy and the sufficiency of the total model, and very often the procedure leads to a very complicated model. Furthermore, a special problem of the traditional approach is to achieve a reasonable description of the short-time variations, which can be especially useful in control situations.

In the proposed method the accuracy of the model is directly estimated and represented as covariance matrices of the states and parameters, and the problem of sufficiency and over-parameterization is readily answered by analyzing the residuals and by considering the variance of the estimates. A model of the important short-time variations for control purposes is obtained, provided that the measurements contain the actual information, i.e., the sampling time is chosen properly.

In an alternative approach the dynamic model is obtained solely by statistical methods. The disadvantage of such a method is the difficulties that might be involved with physical interpretation of the parameters in the most commonly used discrete-time parameterization. In the proposed method the parameterization is kept in continuous time which ensures a more reasonable interpretation of the parameters. Furthermore, the continuous-time formulation makes it possible to change the sampling time properly.

In the considered experiment it turns out that a model with two time constants gives a reasonable description of the observed variation. A model is formulated which, probably in many situations, can serve as a reasonable approximation of the heat dynamics of buildings where the heat capacity in the outer wall is of minor importance. Compared to the single-time-constant model frequently used, the proposed model is an extension containing two time constants, and thus it is able to describe both the long- and the short-time variations of the indoor air temperature.

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