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Identification of Transfer Functions for Control of Greenhouse Air Temperature

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The purpose of this paper is to describe a method for determining transfer functions for the air temperature in greenhouses. Such models are required for using modern control strategies such as the generalized predictive controller. The input variables are heat from the heating system and from solar radiation. The structure of the transfer functions is identified from preliminary estimates of the impulse response functions obtained by using ridge regression.

In order to provide reliable and accurate parameter estimates an experiment was designed and conducted in January and February 1991. Data were sampled at 2 min intervals over two periods. During the experiment the heat supply was controlled by a pseudo random binary signal (PRBS) in order to avoid correlation between the heat supply and other variables, and in order to ensure that the dynamic characteristics of the greenhouse were present in the data.

For both periods, a transfer function model was determined, and good agreement between the models for the two periods was observed. The methods used show that for the greenhouse considered, a third order model with two time constants for the response from the heating system is adequate. A short time constant of about 5 min and a longer time constant of about 25 min were found.

1. Introduction

The energy input in greenhouses in a temperate climate zone as in Denmark is normally large, especially in winter. In order to reduce the energy consumption in greenhouses, research has been done to improve the system for controlling the supply of energy. Today, energy supply in greenhouses is controlled typically by a proportional integral differential (PID) controller.

The long-term purpose of the project is to devise a control system whereby the energy supply is controlled by a prediction of temperature states in the greenhouse. Controllers of this type are the minimum variance controller[1] and the general predictive controller.[2] Prediction-based controllers have proved to be powerful[3] and they contain the ordinary PID controllers as special cases. Model-based adaptive control of greenhouses has previously been considered by Udink ten Cate[4] and Davis and Hooper.[5] The performance of prediction-based control systems depends on the possibility of obtaining good predictions of the temperature states in the greenhouse. The prediction of the temperature states depends on factors that have a significant influence on the air temperature inside the greenhouse. As a first step in the project an experiment was carried out to develop a method for identifying the dependence of the air temperature inside the greenhouse on the heat input from the heating system and solar radiation. This paper describes the results of this experiment.

2. The Box–Jenkins transfer function model

The dependence of the air temperature inside the greenhouse on the heat input from the heating system and the solar radiation will be described by a Box–Jenkins transfer function model,[6] which is briefly introduced in this section.

A time series is a set of observations generated sequentially in time. Let $\{y_t\} = y_1, y_2, \ldots, y_n$ denote a time series of length n with equidistant sampling interval. It is convenient to introduce the backward shift operator, B, defined by $By_t = y_{t-1}$.

Let $\{x_t\} = x_1, x_2, \ldots, x_n$ be another time series

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sampled at the time intervals as $\{y_t\}$. On the assumption that the relationship between $\{x_t\}$ and $\{y_t\}$ is a causal, linear and time invariant system, then there exists an impulse response function $\{v_k\}$ so that y_t is given by the convolution between v_k and x_t (Ref. 6).

$$y_t = \sum_{k=0}^{\infty} v_k x_{t-k} \tag{1}$$

Using the backward shift operator, B, this is written

$$y_t = v(B)x_t \tag{2}$$

Where $v(B) = v_0 + v_1 B + v_2 B^2 + \cdots$. The function $v(B)$ is often approximated by a transfer function which is a ratio between two polynomials[6]

$$y_t = \left(\frac{\omega_0 - \omega_1 B - \omega_2 B^2 - \cdots - \omega_s B^s}{1 - \delta_1 B - \delta_2 B^2 - \cdots - \delta_r B^r}\right) B^b x_t \tag{3}$$

where (r, s) is the order of the transfer function, $\omega_0, \omega_l, \ldots, \omega_s, \delta_1, \delta_2, \ldots, \delta_r$ are constants and b is a delay from the input series $\{x_t\}$ to the output series $\{y_t\}$.

Most frequently, however, not all the variation of $\{y_t\}$ can be described by $\{x_t\}$. In order to take this into account the model is rewritten as

$$y_t = \left(\frac{\omega_0 - \omega_1 B - \omega_2 B^2 - \cdots - \omega_s B^s}{1 - \delta_1 B - \delta_2 B^2 - \cdots - \delta_r B^r}\right) B^b x_t + N_t \tag{4}$$

where $\{N_t\}$ is a possibly autocorrelated time series.

On the assumption that the variation of $\{N_t\}$ can be described by an auto regressive moving average model (ARMA model) then the Box–Jenkins[6] transfer function model is finally obtained, i.e.

$$y_t = \left(\frac{\omega_0 - \omega_1 B - \omega_2 B^2 - \cdots - \omega_s B^s}{1 - \delta_1 B - \delta_2 B^2 - \cdots - \delta_r B^r}\right) B^b x_t + \left(\frac{1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q}{1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p}\right) \varepsilon_t \tag{5}$$

where $\theta_1, \theta_2, \ldots, \theta_q, \phi_1, \phi_2, \ldots, \phi_p$ are constants (p, q) is the order of the ARMA model of $\{N_t\}$ and $\{\varepsilon_t\}$ is a white noise process with variance σ_ε^2. The model may be written in short as

$$y_t = \frac{\omega(B)B^b}{\delta(B)} x_t + \frac{\theta(B)}{\varphi(B)} \varepsilon_t \tag{6}$$

Frequently, it is required to extend the Box–Jenkins transfer function model to a model with several input variables, i.e.

$$y_t = \sum_i \frac{\omega_i(B)B^{b_i}}{\delta_i(B)} x_{i,t} + \frac{\theta(B)}{\varphi(B)} \varepsilon_t \tag{7}$$

where $x_{i,t}$ is the ith input. Note, that a transfer function component is introduced for each input.

3. The experiment

The experiment was conducted from 28 January to 10 February 1991. This part of the year was chosen because a typical winter period with a typical demand on heat supply was wanted. The greenhouse is smaller than commercial greenhouses, being 8 m × 21·5 m in base area. It is constructed of steel with glass in aluminium frames, where the distance between the frames is 0·8 m. The greenhouse is placed in an area with other greenhouses used for experiments. A more detailed description of the house can be found in Nielsen *et al.*[7]

Inside the greenhouse there were four benches of which two were half filled with plants. The heating system consisted of three pipe heating systems, with horizontal pipes running the length of the greenhouse with heat provided by hot water running through the pipes. The largest heating system is placed towards the top of the house, along the side wall, and is called the wall/top heating system. The second and smallest system is placed just under the bench. The third heating system is placed near the floor, and is called floor heating.

During the experiment, a two-level strategy of heating from the heating system was used. The actual supply, either low or high, was determined by a pseudo random binary signal (PRBS). The two levels were obtained in the following way. The flow of water was kept constant, and the inlet water temperature for the heating system just under the benches was kept constant. The inlet water temperature for the other two systems, i.e. the wall/top and floor heating, were the same and controlled by the PRBS, which then determined whether the heat supply should be on the high or low level. In order to obtain the two levels, the difference between the inlet temperature and the air temperature was kept at one level for the low heat supply and a higher level for the high heat supply.

3.1. *The pseudo random binary signal (PRBS)*

When a greenhouse is heated, the control system will introduce a correlation between solar radiation and energy supplied by the heating system. This happens because the control system attempts to maintain a constant air temperature. The correlation introduces a difficulty at the identification stage. To avoid the correlation between the heat input and other input variables, the energy from the heating system was controlled by a pseudo random binary signal (PRBS).[8] The signal determines the switch between the two levels of energy input into the greenhouse. The advantage of using a PRBS is that it has an autocorrelation close to white noise and hence is not correlated with other input signals.

A PRBS is a deterministic signal generated by a shift register which switches between two levels, zero and one. Most frequently, the PRBS signal is constructed by selecting the shortest (λ) and the longest time interval ($n\lambda$) where the signal is constant. In this experiment the choice of the two intervals was $\lambda = 72$ min for the shortest time interval and $4\lambda = 288$ min for the longest interval. The complete period of the signal is of $\lambda(2^n - 1) = 1080$ min (18 h). In *Fig. 1* one complete period of the PRBS is shown. Since the PRBS is a deterministic signal, the signal is repeatable. See Godfrey[8] for further details about PRBS.

3.2. *Data and data sampling*

The experiment was divided into two periods, namely, midnight on 28 January to midnight on 30 January and midnight on 7 February to midnight on 10 February 1991. During each period the temperature inside the greenhouse, the solar radiation, the temperature outside the greenhouse and inlet and outlet water temperature of the heating system were sampled every second minute. All the data was collected by a PDP 11 computer.

The air temperature in the greenhouse (T_i) was measured in the middle of the house 1·5 m above the floor and behind an aspirated screen with a platinum resistance temperature sensor.

The temperature in the heating system was measured with paired platinum resistances temperature sensors. A sensor was placed for measuring the inlet water temperature (T_1) of water entering the heating system. The outlet water temperature (T_2) was measured by another sensor placed on the heating system where the water leaves the house. The solar radiation (Q_s) was measured with a Kipp & Zone solarimeter placed on the top of the roof on a neighbouring greenhouse. The outdoor temperature (T_0) was measured in a Stevenson Screen.

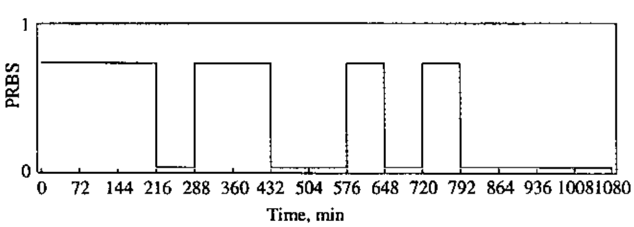

Fig. 1. One period (18 h) of the used pseudo random binary signal (PRBS)

4. Results

In this paper, the energy input from the heating system is used as an input variable for prediction of the air temperature. The energy input of the heating system to the greenhouse is assumed to be proportional to a mean temperature difference (T_d) which is the difference between the mean temperature of the pipes in the heating systems and the air temperature (T_i) in the greenhouse. The formula used is

$$Q = cT_d \tag{8}$$

$$T_d = \tfrac{1}{2}(T_1 + T_2) - T_i \tag{9}$$

where Q is the energy input to the greenhouse, and c is a constant belonging to the heating system.

Data from 1 day of the experiment are shown in *Fig. 2.* Solar radiation (Q_s) is the radiation on a horizontal surface outside the house. The theoretical maximum of the solar radiation for the period when the experiment was running is about 350 W/m². The day shown in *Fig. 2* is typical for a day with an overcast sky.

The plot of T_d in *Fig. 2* shows how PRBS determines the mean temperature difference. When the PRBS was 1 the mean temperature difference rises about 13°C compared with the situation where the signal was zero. A higher mean temperature

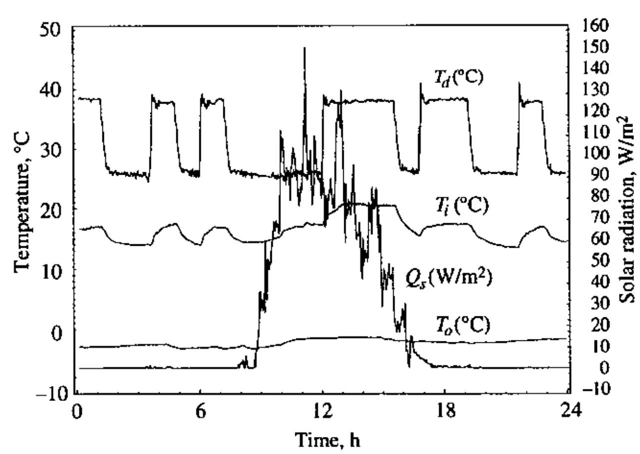

Fig. 2. The mean temperature difference (T_d), air temperature (T_i), solar radiation (Q_s) and outdoor temperature (T_o) on 7 February 1991

difference gives a higher input of energy into the greenhouse and results in an increase of the air temperature (T_i). Also, the solar radiation which increases before and after midday gives a higher air temperature.

4.1. *Spectral analysis*

Often, spectral analysis is useful as a diagnostic tool in the analysis of time series.[6,9,10]

Fig. 3 shows the spectrum of the air temerature. It is seen that the variation is dominated by low frequencies. White noise is characterized by having a constant spectrum for all frequencies. Hence white noise behaviour is seen for frequencies larger than roughly 0·2 cycles per 2 min. Compared with the 95% confidence interval it can be concluded that the main variability in the time series is at low frequencies and concentrated at cycles where the period is larger than 2 min/0·2 = 10 min.

4.2. *Identification of models with ridge regression*

In order to estimate the transfer function component in Eqn (7), knowledge of the values of s, r and b from each of the input variables is needed. Box and Jenkins[6] suggested that these values can be guessed from the pattern of preliminary estimates of the impulse response weights $\{v_k\}$ of Eqn (1). They also proposed a preliminary estimations procedure called prewhitening procedure. But in the present case the procedure is not useful because it requires ARMA models of the input variable but the heat energy from the heating system cannot be described by an ARMA model because it is switching between two levels due to the control by the PRBS.

Hence, another method has been used, namely, that of Edlund[11] who proposed a preliminary estimation procedure based on ridge regression. *Figs 4* to *7* show the preliminary estimates of impulse response weights, $\hat{v}_{T,k}$ and $\hat{v}_{Q,k}$ (the "hat" indicates that the values are estimates) of the model

$$T_{i,t} = \sum_{k=0}^{99} (v_{T,k} T_{d,t-k} + v_{Q,k} Q_{s,t-k}) + N_t \qquad (8)$$

where $v_{T,k}$ is the impulse response weight of heat from the heating system at the kth previous measurement, and $v_{Q,k}$ is the impulse response weight of solar radiation at the kth previous measurement. Note that only the first 100 values of the impulse response weights are considered.

An important item of information from the preliminary estimates of the impulse weights in *Figs 4* to *7* is the immediate response already at $k = 1$. Therefore, any time delay between air temperature and the dependent factors, seems to be at most one sample (2 min). At the same time, it is important to remember that the pattern of the impulse response weights where k is larger than about 50 are of less importance compared with the first impulse response weights.[10] Especially for $k > 90$, the weights possessed a different pattern compared with impulse response weights for $k < 90$. For the values of k near 100 the results are simply not reliable, since the preliminary estimates at these impulse weights try to account for the influence

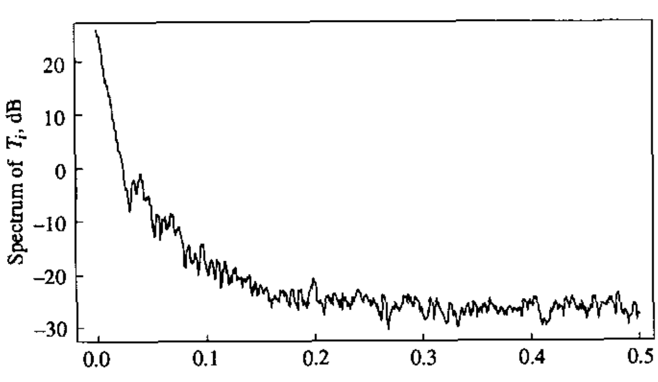

Fig. 3. Spectra of air temperature, 95% confidence limit is (−2·39, 3·31) dB

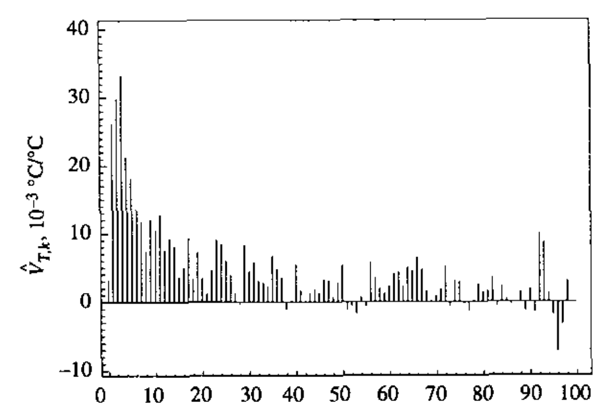

Fig. 4. Preliminary estimates of the impulse response weights obtained from ridge regression of T_i on the previous measurements of T_d in period 28–30 January 1991

of weights for $k > 100$, which are not considered in the ridge regression.

Preliminary estimates of the impulse response function from the energy from the heating system seem to show a peak at $k = 3$ in the first period (*Fig. 4*). If the impulse response weights from $k > 3$ can be described by an exponential decay, then the denominator of the transfer function component from the energy of the heating system must contain a model of first order.[6] The impulse weights for $k < 3$ follow no fixed pattern. These weights will determine the order of the numerator in the transfer function. However, due to the small coefficient at $k = 0$, only three values in the numerator polynomial are needed. Hence, the order of the numerator polynomial is two. A reasonable transfer function component [see Eqn (3)] relating the energy from the heating system to the air temperature might be:

$$v(B) = \left(\frac{\omega_0 - \omega_1 B - \omega_2 B^2}{1 - \delta_1 B}\right) B \tag{9}$$

The impulse response weights of the heating system in *Fig. 5* does not seriously contradict the above model in Eqn (9).

In *Figs 4* and *5* the pattern of the coefficients shows some evidence of a combination of an exponential decay and an oscillation structure. To handle this oscillation the order of polynomial in the denominator has to be increased by two. Thus, a possible revised model is:

$$v(B) = \left(\frac{\omega_0 - \omega_1 B - \omega_2 B^2}{1 - \delta_1 B - \delta_2 B^2 - \delta_3 B^3}\right) B \tag{10}$$

In the same way *Figs 6* and *7* can be analysed for the structure of the transfer function from solar radiation to the air temperature. The outcome of this analysis is that no order bigger than three is expected for the numerator and the denominator of the transfer function from the solar radiation.

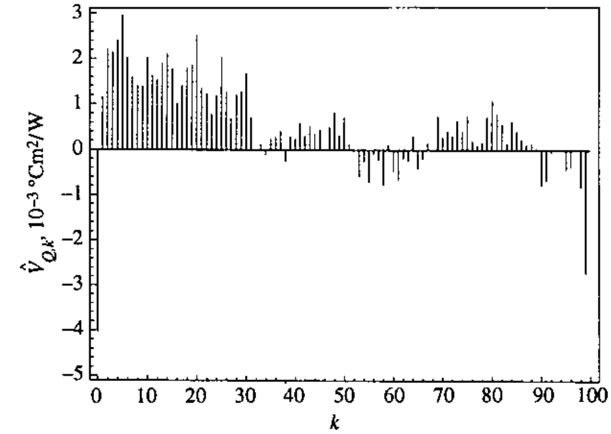

Fig. 6. Preliminary estimates of the impulse response weights obtained from ridge regression of T_i on the previous measurements of Q_s in period 28–30 January 1991

For simplicity, a third order polynomial is considered for both numerators, and no time delay from both inputs is assumed. Hence the following preliminary overall model for the air temperature of the greenhouse is considered to be

$$\begin{aligned} T_i = & \frac{\omega_{T,0} - \omega_{T,1} B - \omega_{T,2} B^2 - \omega_{T,3} B^3}{1 - \delta_{T,1} B - \delta_{T,2} B^2 - \delta_{T,3} B^3} T_{d,t} \\ & + \frac{\omega_{Q,0} - \omega_{Q,1} B - \omega_{Q,2} B^2 - \omega_{Q,3} B^3}{1 - \delta_{Q,1} B - \delta_{Q,2} B^2 - \delta_{Q,3} B^3} Q_{s,t} \\ & + \frac{\theta(B)}{\varphi(B)} \varepsilon_t \end{aligned} \tag{11}$$

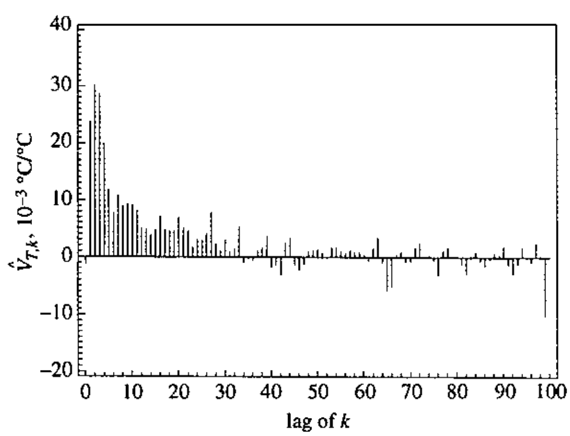

Fig. 5. Preliminary estimates of the impulse response weights obtained from ridge regression of T_i on the previous measurements of T_d in period 7–10 February 1991

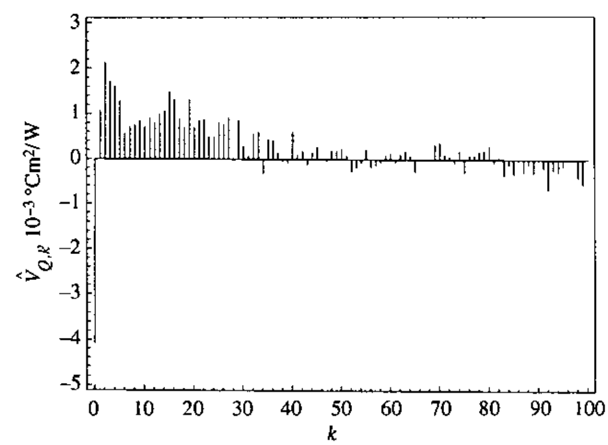

Fig. 7. Preliminary estimates of the impulse response weights obtained from ridge regression of T_i on the previous measurements of Q_s in period 7–10 February 1991

Table 1

Estimates and standard error for parameters of models in period 28–30 January 1991. Mean value (18·9°C) subtracted from the data

						Heating system							Solar radiation								
Model No.	$\hat{\theta}_1$	$\hat{\theta}_2$	ε_t $\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\phi}_3$	$\hat{\omega}_0$	$\hat{\omega}_1$ (10^{-3} °C/°C)	$\hat{\omega}_2$	$\hat{\omega}_3$	$\hat{\delta}_1$	$\hat{\delta}_2$	$\hat{\delta}_3$	$\hat{\omega}_0$	$\hat{\omega}_1$ (10^{-3} °C m^2/W)	$\hat{\omega}_2$	$\hat{\omega}_3$	$\hat{\delta}_1$	$\hat{\delta}_2$	$\hat{\delta}_3$	$\hat{\sigma}^2_\varepsilon$ °C^2	SBC
1	1·34 (0·06)	−0·54 (0·05)	2·04 (0·06)	−1·29 (0·12)	0·24 (0·06)	−1·5 (2·4)	−30·0 (6·2)	20·1 (9·2)	−0·2 (5·5)	1·99 (0·11)	−1·47 (0·18)	0·44 (0·08)	1·62 (0·40)	−1·85 (1·01)	0·16 (1·62)	−1·14 (1·22)	0·64 (0·64)	0·56 (0·78)	−0·26 (0·61)	0·01005	−3668
2	1·19 (0·04)	−0·32 (0·02)	1·89 (0·04)	−0·89 (0·04)			26·5 (1·7)	18·0 (2·2)		2·01 (0·05)	−1·49 (0·08)	0·45 (0·04)		2·98 (0·20)			0·952 (0·005)			0·01016	−3698
3	1·19 (0·03)	−0·30 (0·02)	1·93 (0·02)	−0·93 (0·02)			31·4 (2·3)			0·89 (0·08)	−0·018 (0·07)			2·85 (0·21)			0·951 (0·007)			0·01058	−3624
4	1·19 (0·03)	−0·30 (0·02)	1·92 (0·02)	−0·93 (0·02)			31·8 (1·1)			0·871 (0·006)				2·85 (0·21)			0·951 (0·007)			0·01057	−3634

Table 2

Estimates and standard error for parameters of models in period 7–10 February 1991. Mean value (16·8°C) subtracted from the data

						Heating system							Solar radiation								
Model No.	$\hat{\theta}_1$	$\hat{\theta}_2$	ε_t $\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\phi}_3$	$\hat{\omega}_0$	$\hat{\omega}_1$ (10^{-3} °C/°C)	$\hat{\omega}_2$	$\hat{\omega}_3$	$\hat{\delta}_1$	$\hat{\delta}_2$	$\hat{\delta}_3$	$\hat{\omega}_0$	$\hat{\omega}_1$ (10^{-3} °C m^2/W)	$\hat{\omega}_2$	$\hat{\omega}_3$	$\hat{\delta}_1$	$\hat{\delta}_2$	$\hat{\delta}_3$	$\hat{\sigma}^2_\varepsilon$ °C^2	SBC
1	1·16 (0·07)	−0·29 (0·07)	1·947 (0·07)	−0·98 (0·15)	0·04 (0·07)	3·2 (1·7)	−21·5 (3·8)	7·5 (6·3)	6·3 (4·6)	1·67 (0·13)	−0·96 (0·23)	0·25 (0·10)	0·81 (0·32)	−2·41 (0·51)	−2·33 (1·04)	−0·20 (0·99)	−0·17 (0·54)	0·60 (0·23)	0·41 (0·46)	0·006978	−5982
2	1·12 (0·03)	−0·25 (0·02)	1·91 (0·03)	−0·91 (0·03)			28·2 (1·4)	16·0 (2·4)		1·76 (0·05)	−1·18 (0·08)	0·36 (0·04)		2·25 (0·15)			0·936 (0·0076)			0·006997	−6027
2	1·12 (0·03)	−0·25 (0·02)	1·90 (0·03)	−0·90 (0·03)			28·5 (1·7)	8·1 (4·4)	13·0 (2·3)	1·50 (0·06)	−0·52 (0·05)			2·21 (0·15)			0·935 (0·0077)			0·007007	−6025
4	1·13 (0·03)	−0·25 (0·02)	1·92 (0·02)	−0·92 (0·02)			29·3 (1·7)	−9·6 (2·7)	13·3 (2·0)	0·889 (0·007)				2·27 (0·15)			0·936 (0·0078)			0·007115	−5988

where $\omega_{T,i}$, $\delta_{T,i}$ and $\omega_{Q,i}$, $\delta_{Q,i}$ are the constants in the transfer functions from the heating system and solar radiation respectively. The structure of $\theta(B)$ and $\varphi(B)$ are identified by conventional time series analysis based on the autocorrelation function of N_t (see Box and Jenkins[6] for further information).

4.3. *Model estimation and validation*

After the identification of the model, the structure of the parameters can now be estimated. For the estimation, a maximum likelihood method is used.[12] All parameters were estimated simultaneously including the parameters of the noise model. The standard errors of the parameters in the models were also estimated. Estimates of the parameters from the different models are shown in Tables 1 and 2. The tables contain the results from period 1 and 2, respectively.

The first model in Tables 1 and 2 corresponds to the most complicated model identified in the previous section, Eqn (11). The other models in the tables correspond to the most attractive reductions of this model. However, a lot of other alternative model structures have been considered, but they are not shown since they are not candidates for a reasonable model. To facilitate comparisons of the different models the Schwartz's Bayesian criterion (SBC) is also shown. This is defined by Schwarz[13] as

$$\mathrm{SBC} = n \log \hat{\sigma}^2 + p \log n \tag{12}$$

where n is the number of observations, p is the total number of parameters and $\hat{\sigma}^2$ is the estimated variance of the white noise process in the model. The best model is the one which has the mathematically smallest value of SBC (note that these values are negative).

Model 2 in Tables 1 and 2 is a third order model of the denominator in the transfer function of the energy from the heating system [as in Eqn (10)] and a first order model of the denominator of transfer function from the solar radiation [as in Eqn (9)]. Compared with model 1, this model is, in both cases, better, owing to significantly lower values of SBC. The difference in the number of parameters between the two models is seven. In model 2, ω_0 is zero and a time delay of 2 min is assumed for the energy input from the heating system and the solar radiation.

In Tables 1 and 2, models 3 and 4 respectively, contain a second and a first order polynomial in the denominator of the transfer function from the heating system. Due to the SBC criteria it is concluded that model 2 is the best model in both cases.

From the estimated parameters of the transfer functions it is possible to calculate the roots of the polynomials. From the roots of the denominator polynomial the time constants can be calculated as:

$$\tau = \frac{\Delta t}{\ln|\alpha|} \tag{13}$$

where Δt is the sampling time (2 min), and α is a root of the denominator polynomial. A 95% confidence limit of τ is calculated by a resampling procedure where the (not shown) correlation between δ_1, δ_2 and δ_3 is used. Table 3 shows the time constants with 95% confidence limit of the heat input from the heating system and the solar radiation for the best model, i.e. model 2 in Table 1 and Table 2.

Two time constants were calculated for the response from the heating system in model 2. This is the outcome of one real root and two complex roots. For the two periods the longest time constant was estimated to 30 min and 21 min, and the shortest time constant to 5·5 min and 4·4 min, respectively. However, the time constants for the two periods are of the same magnitude. Thus, values of about 5 and 25 min can be proposed for the short and long time constants, respectively. In Table 3 the estimated constant for the response from the solar radiation was 30 and 40 min for the two periods. These values are not significantly different, as indicated by the 95% confidence interval in Table 3. Hence, a single value of about 35 min can be proposed for the time constant related to the solar radiation.

The impulse response function of the energy from the heating system is calculated from model 2, for both periods and shown in *Figs 8* and *9*. The impulse response function in *Figs 8* and *9* can be compared with the preliminary impulse response estimates in *Figs 4* and *5*. The oscillation structure in the impulse response function is most likely owing to the controller of the heating system itself. The same oscillation structure can also be seen in *Fig. 2*, especially when the heating system is in the beginning of a period with a high level of energy output.

The assumption that $\{\varepsilon_t\}$ is white noise, in Eqn (5), was tested by means of cumulative periodograms.[6] The cumulative periodograms, with a 95% confidence

Table 3
Time constant, τ (min), for model 1 in Table 1 and Table 2

	Period	*Heating system*		*Solar radiation*
τ (min)	28–30 January 1991	30	5·5	40
95% C.I.		[24; 40]	[4·2; 7·4]	[33; 52]
τ (min)	7–10 February 1991	21	4·4	30
95% C.I.		[18; 27]	[3·3; 5·7]	[24; 40]

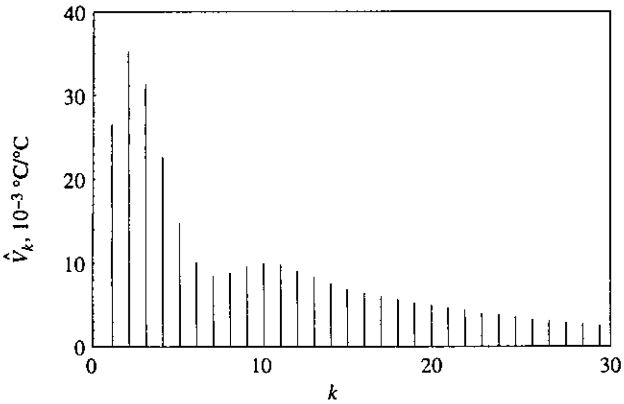

Fig. 8. Impulse response function of the energy from the heating sytem in period 28–30 January 1991

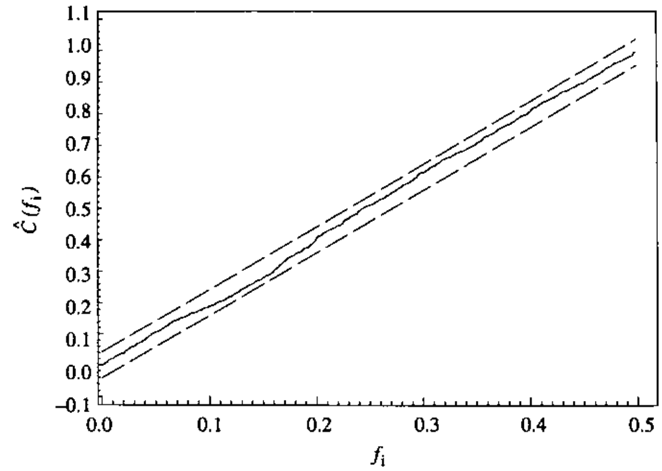

Fig. 10. Cumulated periodogram of residuals of air temperature in period 28–30 January 1991 (with 95% confidence limits)

limit, were calculated for model 2 and are shown in *Figs 10* and *11* for the two periods.

The cumulative periodograms in *Figs 10* and *11* are inside the confidence limits. This means that model 2 seems to give a reasonable description of the variations of the air temperature in the greenhouse.[6]

5. Discussion

The time constants found in this experiment are in accordance with time constants found by Udink ten Cate.[4] He found one dominant time constant of approximately 20 min of the heat transfer from the heating system to the air temperature in a greenhouse. Udink ten Cate[4] also found that the time constant for energy from the solar radiation was larger than the time constant from the heating system. Here it is found that the dominant time constant for energy from the solar radiation is between 30 and 40 min.

The best model for both periods is almost alike. However, the model used does not include the humidity in the greenhouse. The humidity of the air will properly influence the time constants of the greenhouses. But it was not possible to include the humidity as a parameter in the transfer model, because the humidity of the air has a strong correlation with the air temperature. Thus, to include the humidity in the model, a non-linear model has to be considered.

Another parameter not included in the present models is the outdoor temperature. But the outdoor temperature turned out to have no significant effect on the indoor temperature in the second period and only

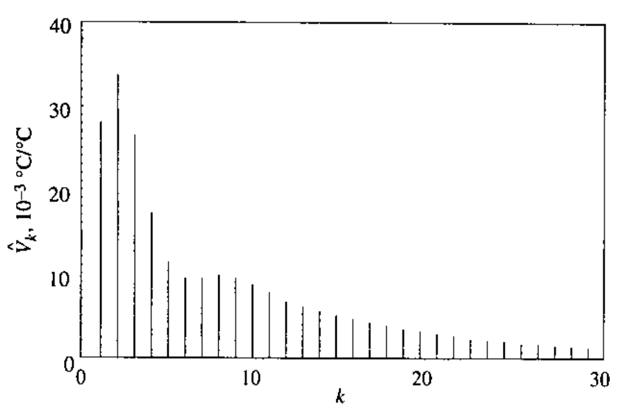

Fig. 9. Impulse response function of the energy from the heating system in period 7–10 February 1991

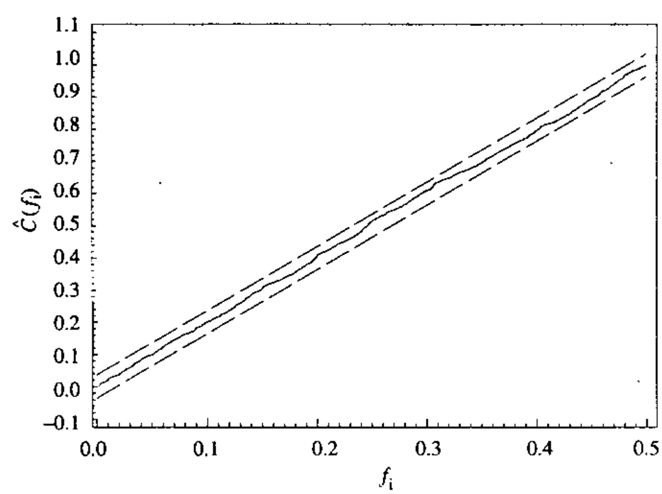

Fig. 11. Cumulated periodogram of residuals of air temperature in period 7–10 February 1991 (with 95% confidence limits)

a small influence, compared with the solar radiation, in the first period. If, however, the outdoor temperature is included in the model as an extra input variable, the estimated model becomes:

$$T_{i(t)} = 18{\cdot}9 + \left(\frac{26{\cdot}9 - 18{\cdot}4B}{1 - 2{\cdot}00B + 1{\cdot}47B^2 - 0{\cdot}44B^3}\right)T_{d(t-1)} + \left(\frac{2{\cdot}72}{1 - 0{\cdot}947B}\right)Q_{s(t-1)} + \left(\frac{0{\cdot}127 + 0{\cdot}053B}{1 - 0{\cdot}719B}\right)T_{o(t-1)} + \left(\frac{1 - 1{\cdot}19B + 0{\cdot}32B^2}{1 - 1{\cdot}86B + 0{\cdot}86B^2}\right)\varepsilon_{(t)}$$

The estimated transfer function from the outdoor temperature is of order one. The root $\alpha_1 = 1{\cdot}39$ corresponds to a time constant of 6·1 min. The estimated variance of residuals is 0·00988, and SBC is estimated to -3736, the decrease of SBC from model 2 (Table 1) to this model is $-3698 - (-3736) = 38$. This shows a significant dependency on the outdoor temperature, but comparing the significant level with the significance level of the solar radiation the outdoor temperature has only a small effect. The reason that the outdoor temperature shows only a small effect in these experiments, is due to a very small variation in the outdoor temperature during the short experiments. If the experiments were run for a longer period or in a climate with large variations in the outdoor temperature, then the temperature would be expected to cause significant variation.

6. Conclusions

A method for identification of transfer function models for the heat dynamics of greenhouses is proposed. By this method it is possible to identify and estimate a reasonable model of the air temperature in a greenhouse. Although a greenhouse is a distributed system, which approximately can be described by a large number of time constants, it has been demonstrated that a reasonable model with only a few time-constants can be found.

The identified models describe the air temperature adequately using only two input variables, namely energy from the heating system and from solar radiation. The identification of the model is based on a two-step procedure. The first step gives preliminary estimates, using ridge regression, of the impulse response function from the solar radiation and the heating system. From the preliminary estimates of the impulse response function the time delay from inputs to output and the number of parameters in the transfer functions are found. In the second step the parameters of the transfer function model are estimated by the method of maximum likelihood.

The model is estimated using data from an experiment. In the experiment the heat supply was controlled by a pseudo random binary signal. By this approach a correlation between the heat supply signal and other input signals is avoided.

From the identified transfer function model the time constants of the system are calculated. The order of the denominator of the transfer function from the solar radiation is estimated to be one. The corresponding time constant is about 35 min.

The order of the denominator in the transfer function from the heating system is estimated to be three. This allows for a pattern of the impulse function with an exponential decay in combination with an oscillation. Two time constants were estimated; a short one of about 5 min and a larger one of about 25 min.

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