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Grey-box modelling of aeration tank settling

Henrik Bechmann[a,b,*], Marinus K. Nielsen[a], Niels Kjølstad Poulsen[b], Henrik Madsen[b]

[a] *Krüger A/S, Gladsaxevej 363, DK-2860 Søborg, Denmark*
[b] *Department of Mathematical Modelling, Technical University of Denmark, Building 321, DK-2800 Lyngby, Denmark*

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Abstract

A model of the concentrations of suspended solids (SS) in the aeration tanks and in the effluent from these during Aeration tank settling (ATS) operation is established. The model is based on simple SS mass balances, a model of the sludge settling and a simple model of how the SS concentration in the effluent from the aeration tanks depends on the actual concentrations in the tanks and the sludge blanket depth.

The model is formulated in continuous time by means of stochastic differential equations with discrete-time observations. The parameters of the model are estimated using a maximum likelihood method from data from an alternating BioDenipho waste water treatment plant (WWTP).

The model is an important tool for analyzing ATS operation and for selecting the appropriate control actions during ATS, as the model can be used to predict the SS amounts in the aeration tanks as well as in the effluent from the aeration tanks.

Keywords: Aeration tank settling; Mass balance; Grey-box models; Statistical identification; On-line measurements

1. Introduction

With the introduction of advanced optimising control systems at waste water treatment plants [1,2] the demand for mathematical models of the important processes in waste water treatment plants is increased. The Aeration tank settling (ATS) principle introduces settling periods in aeration tanks of alternating plants and enables increased amounts of suspended solids (SS) to be stored in the aeration tanks during rain storms. ATS increases the hydraulic capacity of the waste water treatment plant (WWTP), but complicates the prediction of the SS concentration in the effluent from the aeration tanks, compared to dry weather operation. During dry weather operation the aeration tanks are fully mixed, and the SS concentrations in the effluent are equal to the SS concentrations in the tanks, but during ATS operation the effluent comes from an aeration tank where the sludge settles. Hence, the SS concentrations in and out of the aeration tanks are not equal during ATS operation.

To minimize the amounts of SS in the effluent, predictive models of the SS concentrations are needed. In [3], a model of SS in the aeration tanks and in the effluent from these is proposed. The model consists of three sub-models: (1) A simple mass balance model for the SS concentrations in the aeration tanks, (2) a sludge settling model and (3) a model for the SS concentration in the effluent from the aeration tanks.

Vesilind [4,5] proposed a sludge settling velocity model of exponential form. During recent years, several refinements to the original model have been proposed, see e.g. Grijspeerdt et al. [6]; Dupont and Dahl [7]; Ekama et al. [8]. In the proposed models several layers in the settling tank are incorporated to permit the calculation of SS profiles over the tank depth and predict the SS concentrations in the return sludge and in the effluent from the clarifier.

*Corresponding author. DELTA, Venlighedsvej 4, DK-2970 Hørsholm, Denmark.
E-mail address: hbe@delta.dk (H. Bechmann).

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Here, the original Vesilind model combined with a simple suction depth model is used to enable prediction of the SS concentration in the effluent from the aeration tank. In order to make the model applicable for real time control purposes, only two layers of variable height in the aeration tank are considered.

In this paper the grey-box modelling approach is used. A grey-box model is a physically based macroscopic model with stochastic terms to count in uncertainties in model formulation and measurement values. The introduction of stochastic terms enables maximum likelihood estimation of the model parameters. The maximum likelihood method provides estimates of the variance of the parameter estimates, which are used to evaluate the uncertainty of the parameters. Hence, the model in Nielsen et al. [3] is reformulated by means of stochastic differential equations, and the parameters are estimated by a maximum likelihood method. Furthermore, the present paper gives a more detailed description of the model.

2. Dry weather and ATS operation

In an alternating WWTP, the aeration tanks are composed of pairs of interconnected tanks. The waste water is directed to one of the tanks, through the connection between the tanks and out of the second tank. In dry weather situations the tanks are fully mixed to enable optimal nutrient removal. The incoming waste water is directed to an aeration tank with anoxic conditions, and thus with denitrification. The other tank from which the effluent is taken is aerated and is hence a tank with nitrification. Depending on the state of the processes in the aeration tanks, the flow path is changed.

During rain storms ATS operation is activated. When the WWTP is in ATS operation, the aeration scheme is changed so that the influent is directed to an aerobic nitrification tank, and the effluent is taken from an anoxic denitrification tank. When the mixers are switched off in the anoxic tank, settling occurs. When the sludge settles in the tank that discharges to the clarifier, the SS concentration in the effluent is lower than the average concentration in the aeration tank. Hereby more SS can be kept in the aeration tanks compared to dry weather operation at the same time as the SS load to the clarifier is decreased.

It is crucial that as much SS as possible is kept in the aeration tanks during the rain storm and not transported to the clarifier, as an increased SS concentration in the aeration tank effluent will limit the hydraulic capacity of the clarifiers, and thus lead to an SS increase in the effluent to the receiving waters.

By introducing intermediate phases with settling and anoxic conditions in both tanks, the SS concentrations in the effluent from the aeration tanks can be further reduced. By proper control of the flow path and the settling, the SS concentration out of the aeration tanks can be optimized, so that the control does not limit the organic capacity (pollution load capacity in terms of COD or BOD flux) of the plant unnecessarily.

Based on measurements and predictions of the influent flow to the WWTP the ATS operation is activated. The use of flow predictions makes it possible to prepare the plant for the increased storm flow, before the storm water actually enters the WWTP. At Aalborg West WWTP, from where the data used here originates, the influent flow prediction horizon is approximately 1 h. Before the influent flow is increased, the recirculation of sludge from the secondary clarifiers to the aeration tanks is increased. Hereby SS is decreased in the clarifiers and increased in the aeration tanks. Furthermore, the hydraulic load to the aeration tanks and clarifiers is increased. When the storm water arrives at the plant, the recirculation flow is decreased to a lower level.

3. Theory

In Fig. 1 the flow through the aeration tanks and clarifiers is illustrated. The black and grey lines illustrate alternative flow paths through the aeration tanks. The influent flow and the recirculation flow are denoted Q_i and Q_r, respectively. X_{ssi}, X_{ssr} and $X_{ssoutat}$ denote SS concentrations in the influent, the return sludge and the effluent from the aeration tanks to the secondary clarifers. The dynamics of the water amounts in the aerations tanks are not considered, i.e. it is assumed that the flows to and from each of the aeration tanks are the same ($Q_i + Q_r$). Furthermore, the SS concentration in the flow between the two aeration tanks is assumed to be the average SS concentration in the feeding tank. When the feeding tank is fully mixed, this assumption is fulfilled, but when settling occurs it is an approximation.

The mass balance equations for each of the aeration tanks depend on the actual flow path designated f_p. When $f_p = 1$ the influent flow is directed to aeration

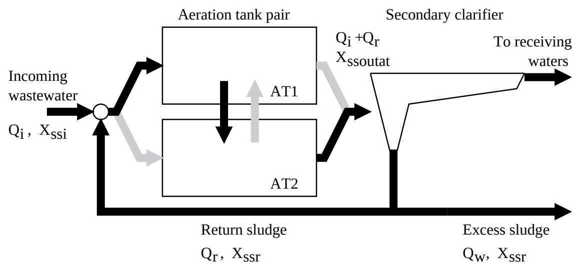

Fig. 1. Flow path through aeration tank pair in an alternating WWTP.

tank 1, and the effluent flow is taken from tank 2. f_p is 0 when the opposite flow path is applied. With V_{at}, X_{ssm1} and X_{ssm2} denoting the volume of each of the equally sized aeration tanks and the average SS concentrations in tanks 1 and 2, respectively, the mass balance equations can be established.

The mass balance equations for the aeration tanks are then

$$\frac{dX_{ssm1}}{dt} = f_p \frac{Q_i X_{ssi} + Q_r X_{ssr} - (Q_i + Q_r) X_{ssm1}}{V_{at}} + (1 - f_p) \frac{(Q_i + Q_r) X_{ssm2} - (Q_i + Q_r) X_{ssoutat}}{V_{at}}, \quad (1)$$

and

$$\frac{dX_{ssm2}}{dt} = f_p \frac{(Q_i + Q_r) X_{ssm1} - (Q_i + Q_r) X_{ssoutat}}{V_{at}} + (1 - f_p) \frac{(Q_i X_{ssi} + Q_r X_{ssr}) - (Q_b + Q_r) X_{ssm2}}{V_{at}}. \quad (2)$$

When mixing is stopped in an aeration tank, the suspended solids settle. A simple two layer model, where the water in the layer above the sludge blanket is assumed to be clear water, and the layer under the sludge blanket is assumed to contain all the SS fully mixed, is used.

The settling velocity for the sludge blanket is modelled according to Vesilind (1968) as

$$\frac{d\,d_{sb}}{dt} = V_0 e^{-n_v X_{sssl}}, \quad (3)$$

where d_{sb} and X_{sssl} denote the sludge blanket depth and the SS concentration in the sludge layer, respectively, see Fig. 2, and V_0 and n_v are sludge volume index (SVI) dependent parameters. For simplicity we use the expressions found by Härtel and Pöpel [9]:

$$V_0 = (17.4e^{-0.0113\mathrm{SVI}} + 3.931)\frac{\mathrm{m}}{\mathrm{h}},$$
$$n_v = (-0.9834e^{-0.00581\mathrm{SVI}} + 1.043)\frac{1}{\mathrm{g}}. \quad (4)$$

If sludge blanket depth measurements are available, V_0 and n_v can be estimated.

As the volume of the sludge layer is $(d_{at} - d_{sb})V_{at}/d_{at}$, the average SS concentration in the sludge layer is

$$X_{sssl} = \frac{d_{at}}{d_{at} - d_{sb}} X_{ssm}, \quad (5)$$

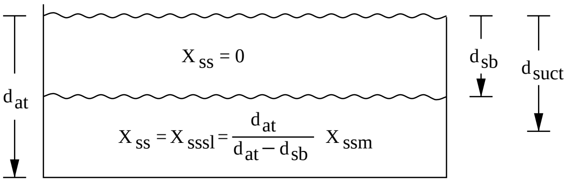

Fig. 2. Two layer model of settling in an aeration tank.

where X_{ssm} is the average SS concentration in the aeration tank.

When the tank is fully mixed, the sludge blanket depth is 0. When mixing is switched on d_{sb} tends towards zero, which is modelled by

$$\frac{d\,d_{sb}}{dt} = -\frac{1}{\tau_{mix}} d_{sb}, \quad (6)$$

where τ_{mix} is a mixing capacity dependent time constant.

Introduce the mixing signals m_1 and m_2 for aeration tanks 1 and 2, respectively. The mixing signals are 1 when the corresponding aeration tank is mixed and 0 otherwise. The signals can then be used to combine the settling equation (3) with the mixing equation (6) for each of the aeration tanks:

$$\frac{dd_{sb1}}{dt} = l(m_1)\left(-\frac{1}{\tau_{mix}} d_{sb1}\right) + (1 - l(m_1)) V_0 e^{-n_v X_{sssl1}} \quad (7)$$

and

$$\frac{dd_{sb2}}{dt} = l(m_2)\left(-\frac{1}{\tau_{mix}} d_{sb2}\right) + (1 - l(m_2)) V_0 e^{-n_v X_{sssl2}}. \quad (8)$$

Here, the aeration tank number is introduced on the sludge blanket depth and average SS concentration variables so that d_{sb1}, d_{sb2}, X_{sssl1} and X_{sssl1} designates the sludge blanket depths and average SS concentrations in aeration tanks 1 and 2, respectively.

The SS concentration in the effluent from an aeration tank is modelled as a function of the suction depth, d_{suct} and the SS concentration in the sludge layer:

$$X_{ssoutat} = \begin{cases} \frac{d_{suct} - d_{sb}}{d_{suct}} X_{sssl} & \text{for } d_{suct} \geqslant d_{sb}, \\ 0 & \text{otherwise.} \end{cases} \quad (9)$$

The suction depth is expected to depend on the flow. However, the flow dependency is not expected to be linear. Hence the suction depth is modelled as

$$d_{suct} = d_0 \left(\frac{Q_i + Q_r}{Q_0}\right)^{b_{suct}}, \quad (10)$$

where d_0 and b_{suct} are positive parameters and $Q_0 = 1000\ \mathrm{m^3/h}$ is a normalization constant. Combining (5) and (9) yields

$$X_{ssoutat} = \frac{d_{suct} - d_{sb}}{d_{suct}} \frac{d_{at}}{d_{at} - d_{sb}} X_{ssm} = \frac{1 - d_{sb}/d_{suct}}{1 - d_{sb}/d_{at}} X_{ssm} \quad \text{for } d_{suct} \geqslant d_{sb}. \quad (11)$$

When $d_{suct} > d_{sb}$ and assuming that $d_{sb} < d_{at}$ and $0 < d_{suct} < d_{at}$, it can be shown that $X_{ssoutat} < X_{ssm}$. This means that during ATS the SS concentration in the outflow from the aeration tanks will be less than the SS concentration in the feeding aeration tanks.

To enable smooth changes in $X_{ssoutat}$ when the point $d_{suct} = d_{sb}$ is passed a smooth threshold function is

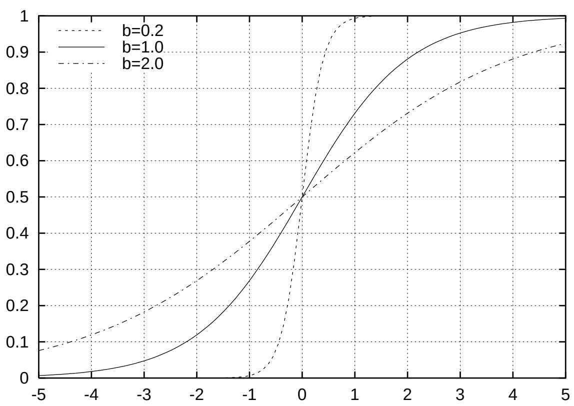

Fig. 3. The logistic function $l(x, a, b)$ for $a = 0$ and different b values.

introduced. Here, the logistic function

$$l(x) = l(x, a, b) = \frac{1}{1 + e^{a - x/b}} \tag{12}$$

is used. For $x = a$ the logistic function is 0.5, i.e. the value of a determines the midpoint of the switch between 0 and 1. By appropriate selection of a and b the change between 0 and 1 of $l(x, a, b)$ can be controlled. In Fig. 3 the logistic function is shown for $a = 0$ and 3 different values of b. In the following $b > 0$ is assumed.

The logistic function (12) is used to calculate X_{ssoutat} while the flow path variable is used to select the discharge tank:

$$\begin{aligned} X_{\text{ssoutat}} = & f_{\text{p}}\left(l(d_{\text{suct}} - d_{\text{sb2}}) \frac{1 - d_{\text{sb2}}/d_{\text{suct}}}{1 - d_{\text{sb2}}/d_{\text{at}}} X_{\text{ssm2}} \right) \\ & + (1 - f_{\text{p}})\left(l(d_{\text{suct}} - d_{\text{sb1}}) \frac{1 - d_{\text{sb1}}/d_{\text{suct}}}{1 - d_{\text{sb1}}/d_{\text{at}}} X_{\text{ssm1}} \right). \end{aligned} \tag{13}$$

As d_{suct} is only dependent on the flow, there is no need to consider different suction depths for each of the aeration tanks.

In order to use a matrix notation, introduce the state vector $\boldsymbol{X}$, the input vector $\boldsymbol{U}$ and the observation vector $\boldsymbol{Y}$:

$$\begin{aligned} \boldsymbol{X} &= [X_{\text{ssm1}}, X_{\text{ssm2}}, d_{\text{sb1}}, d_{\text{sb2}}]', \\ \boldsymbol{U} &= [f_{\text{p}}, m_1, m_2, X_{\text{ssr}}, Q_{\text{i}}, Q_{\text{r}}]', \\ \boldsymbol{Y} &= [X_{\text{ssm2}}, X_{\text{ssoutat}}]'. \end{aligned} \tag{14}$$

Here, it is assumed that aeration tank 2 is equipped with a suspended solids sensor.

By use of the vector function $\boldsymbol{f}(\boldsymbol{X}, \boldsymbol{U}, t)$ the mass balances and sludge blanket depth equations can be expressed in a vector differential equation:

$$\frac{d\boldsymbol{X}(t)}{dt} = \boldsymbol{f}(\boldsymbol{X}, \boldsymbol{U}, t), \tag{15}$$

where $\boldsymbol{f}(\boldsymbol{X}, \boldsymbol{U}, t)$ is easily constructed from Eqs. (1)–(8) and (14).

The measurements are described by the observation equation

$$\boldsymbol{Y}(t) = \boldsymbol{h}(\boldsymbol{X}, \boldsymbol{U}, t), \tag{16}$$

where $\boldsymbol{h}(\boldsymbol{X}, \boldsymbol{U}, t)$ is constructed from Eqs. (13) and (14).

To take into account uncertainties in the model formulation and to enable use of the maximum likelihood parameter estimation method, stochastic noise terms are introduced. Hence, Eq. (15) turns into a stochastic differential equation, where the continuous time equations describing the mass balances and the sludge blanket depths in the aeration tanks can be written as the so-called Itô differential equation [10]:

$$d\boldsymbol{X}(t) = \boldsymbol{f}(\boldsymbol{X}, \boldsymbol{U}, t)\, dt + d\boldsymbol{w}(t), \tag{17}$$

where the stochastic process $\boldsymbol{w}(t)$ is assumed to be a vector Wiener process (see e.g. [11]), with covariance

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_{\text{ss}}^2 & 0 & 0 & 0 \\ 0 & \sigma_{\text{ss}}^2 & 0 & 0 \\ 0 & 0 & \sigma_{\text{sb}}^2 & 0 \\ 0 & 0 & 0 & \sigma_{\text{sb}}^2 \end{bmatrix}. \tag{18}$$

The observation uncertainties are included in the observation equation

$$\boldsymbol{Y}(t) = \boldsymbol{h}(\boldsymbol{X}, \boldsymbol{U}, t) + \boldsymbol{e}(t), \tag{19}$$

where the term $\boldsymbol{e}(t)$ is the measurement error, which is assumed to be a zero mean Gaussian white noise sequence independent of $\boldsymbol{w}(t)$ and with covariance matrix

$$V(\boldsymbol{e}(t)) = \begin{bmatrix} \sigma_{\text{ss2}}^2(t) & 0 \\ 0 & \sigma_{\text{ssoutat}}^2(t) \end{bmatrix}. \tag{20}$$

4. Estimation method

The method used to estimate the parameters of the model (17) and (19) is a maximum likelihood method for estimating parameters in stochastic differential equations based on discrete-time data given by (19). For a more detailed description of the method refer to Bechmann et al. [12], Madsen and Melgaard [13] or Melgaard and Madsen [14]. The applied maximum likelihood method enables estimation of the uncertainties of the parameter estimates, and provided that the model is correct, the parameter estimates are central.

It is well known that maximum likelihood estimates are efficient, and furthermore this estimation frame work enables a rich family of test possibilities. As an example it is straight forward to test for model alternatives.

Unreliable measurements are handled by adjusting the variance of $\boldsymbol{e}(t)$. When an unreliable observation is encountered, the corresponding observation variance is considerably increased. As the extended Kalman filter included in the estimation method uses the observation noise variance and the intensity of the Wiener process, that count for state noise, the unreliable observations are taken into account by the estimation procedure.

5. Results and discussion

Aalborg West WWTP, from where the data used for the estimations originates, is a 330,000 PE activated sludge plant, with three aeration tank pairs, which are controlled in an identical way, except for a time delay between the tank pairs. The master tank pair consists of aeration tanks 5 and 6, of which tank 6 is equipped with an SS sensor. The flow path and mixing of tanks 3 and 4 are delayed $T_d = 12$ min in relation to tanks 5 and 6, and tanks 1 and 2 are delayed further T_d. To include all six aeration tanks in the model, it is extended with equations for the four additional tanks. The flows to and from each tank pair are reduced to a third of the total flows, and the time delay between the tank pairs is taken into account in the flow path and mixing signals to the respective tank pairs. The resulting SS concentration out of the aeration tanks is the average of the SS concentrations out of the three tank pairs. The Aalborg West WWTP model is thus a 12 state non-linear model with two observations, X_{ssm6} and X_{ssoutat}.

The measurements of the average SS concentration in aeration tank 6 are only reliable when the tank is fully mixed. This is taken into account by adjusting the variance of $\boldsymbol{e}(t)$ according to the mixing of aeration tank 6. When the tank is not mixed, $\sigma^2_{\mathrm{ss6}}(t)$ is large (ideally ∞) compared to the value of $\sigma^2_{\mathrm{ss6}}(t)$ when the tank is mixed.

Aalborg West WWTP is equipped with a Superiour Tuning And Reporting (STAR) control system [1,2], which optimizes the operation of the plant. In the STAR system, measurements are fetched and control actions are computed every 6 min. It is good practice to select a sampling time which reflects the dynamics or time constants of interest. It is, for instance, well known that a model estimated using a given sampling time is optimal for one-step ahead predictions. Hence in order to obtain a model with a reasonable prediction performance the data was resampled to a longer sample period. Furthermore, it turned out that there was a significant time delay between the input variables (the flow direction, the mixing signals and the flows and concentrations to the aeration tanks) and the output variables (SS concentrations in aeration tank 6 and out of the aeration tanks). The time delay is caused by the sludge settling process, and was found to be 0.8 h. The new sampling interval was selected to 0.2 h.

The parameters of the model were estimated on one data set and the resulting model was cross validated on another data set. During ATS Q_{i} varies between approx. 4000 $\mathrm{m^3/h}$ and approx. 11,000 $\mathrm{m^3/h}$ and Q_{r} varies between approx. 1000 and 2500 $\mathrm{m^3/h}$.

It was not possible to estimate the sludge settling model parameters V_0 and n_{v} and the parameters of the suction depth model d_0 and b_{suct} simultaneously. This is due to the fact that these two sub-models are closely correlated, as a change in the sludge settling model will be compensated by an equivalent change in the suction depth model. The SVI for the estimation data set was 142, hence the sludge settling parameters found from (4) are

$$V_0 = 7.43 \ \mathrm{m/h} \quad \text{and} \quad n_{\mathrm{v}} = 0.612 \ \mathrm{m^3/kg\ SS}. \tag{21}$$

The SVI for the validation data set was 177, which gives the corresponding sludge settling parameters

$$V_0 = 6.29 \ \mathrm{m/h} \quad \text{and} \quad n_{\mathrm{v}} = 0.691 \ \mathrm{m^3/kg\ SS}. \tag{22}$$

By inspecting the data before the estimation was carried out, systematic errors in the SS concentration measurements were observed. As it is not possible from the available measurements to detect which of the measurements that are correct, it was decided to use X_{ss6} as the reference. The errors on the measurements of X_{ssoutat} were included in the model as offset errors, even though other methods could be applied. For the errors in the return sludge measurements both an additive and a multiplicative form were tried out. It was found that both types gave similar results, and the multiplicative form was used in the final estimations. The bias on X_{ssoutat} designated $X_{\mathrm{ssout,b}}$ and the factor on X_{ssr} designated $X_{\mathrm{ssr,f}}$ were estimated simultaneously with the other parameters.

The influent SS concentration X_{ssi} was sought estimated as constant during the period considered. This parameter was, however, found to be insignificant, and therefore excluded from the final estimation.

The estimated parameters as well as their estimated standard deviations are shown in Table 1. All the parameters except $\sigma^2_{\mathrm{ssoutat}}$ are estimated with small

Table 1
Maximum likelihood estimates of the parameters of the model

Parameter	d_0	b_{suct}	$X_{\mathrm{ssr,f}}$	$X_{\mathrm{ssout,b}}$	σ^2_{ss}	σ^2_{sb}	σ^2_{ss6}	$\sigma^2_{\mathrm{ssoutat}}$
Unit	m	—	—	g/l	$(\mathrm{g/l})^2$	$\mathrm{m^2}$	$(\mathrm{g/l})^2$	$(\mathrm{g/l})^2$
Estimate	1.005	0.164	1.129	0.143	0.0259	1.34	$1.91\ 10^{-4}$	$1.78\ 10^{-7}$
Standard deviation	0.015	0.022	0.009	0.014	0.0038	0.09	$0.28\ 10^{-4}$	$2.08\ 10^{-7}$

standard deviations. The estimate of $\sigma^2_{\mathrm{ssoutat}}$ is thus uncertain.

The measured and modelled SS concentrations are shown in Figs. 4 and 5, for a part of the estimation data set and the validation data set, respectively. The mixing signal for aeration tank 6 is included in the X_{ss6} graphs to indicate the validity of the X_{ss6} measurements, as these are only reliable when mixing is on. Note that the modelled SS concentrations are simulations based only on the input variables to the model, and not one-step ahead predictions, which use the measurements of the output variables at every time step to predict the output at the next time step.

From Figs. 4 and 5 it is clear that the SS concentrations in the mixed outflow of the three aeration tank pairs to the secondary clarifier are systematically lower than the SS concentrations in the aeration tanks. This is due to the ATS operation, where the sludge is settling in the aeration tanks that feed the secondary clarifier.

For the validation data set the V_0 and n_{v} parameters for both SVI = 142 (the estimation data set value) and SVI = 177 (the validation data set value) were tried. The

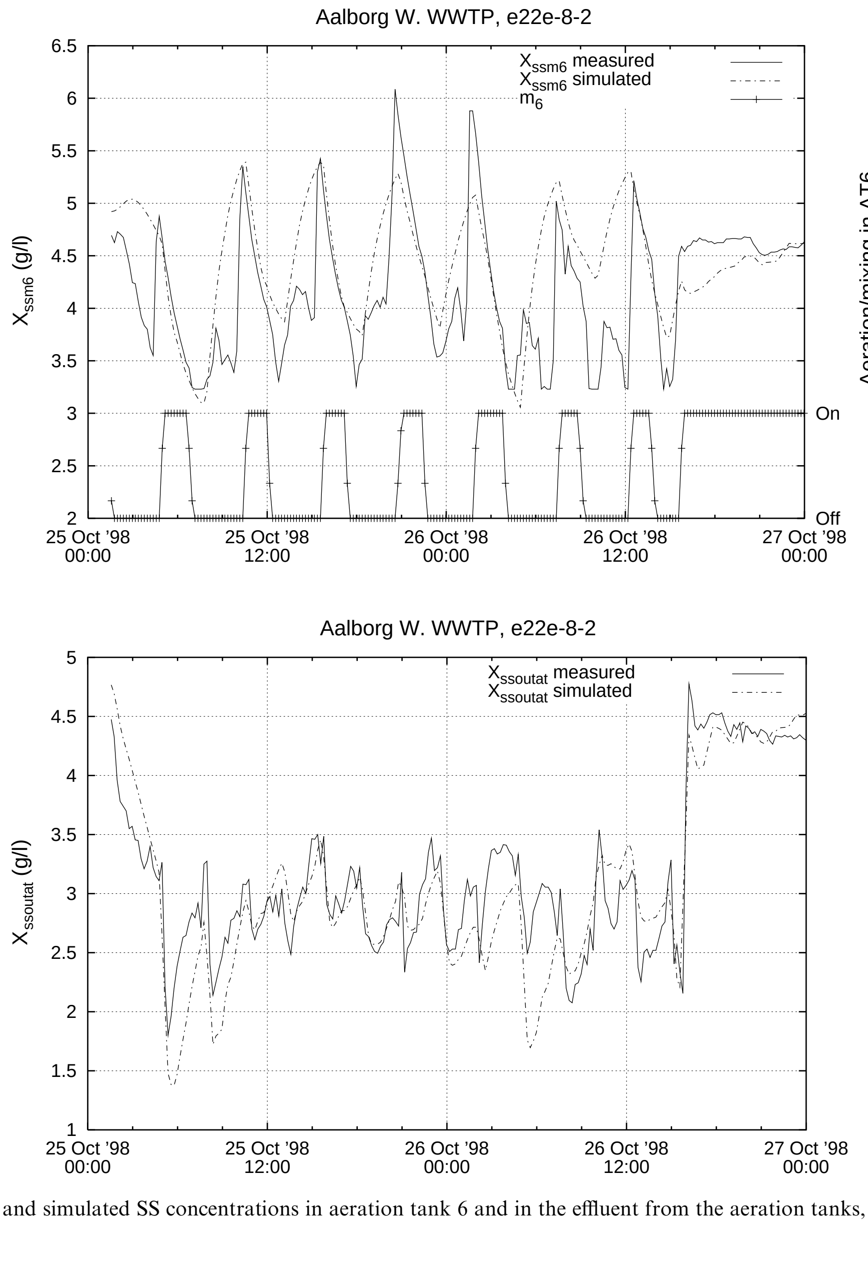

Fig. 4. Measured and simulated SS concentrations in aeration tank 6 and in the effluent from the aeration tanks, estimation data set.

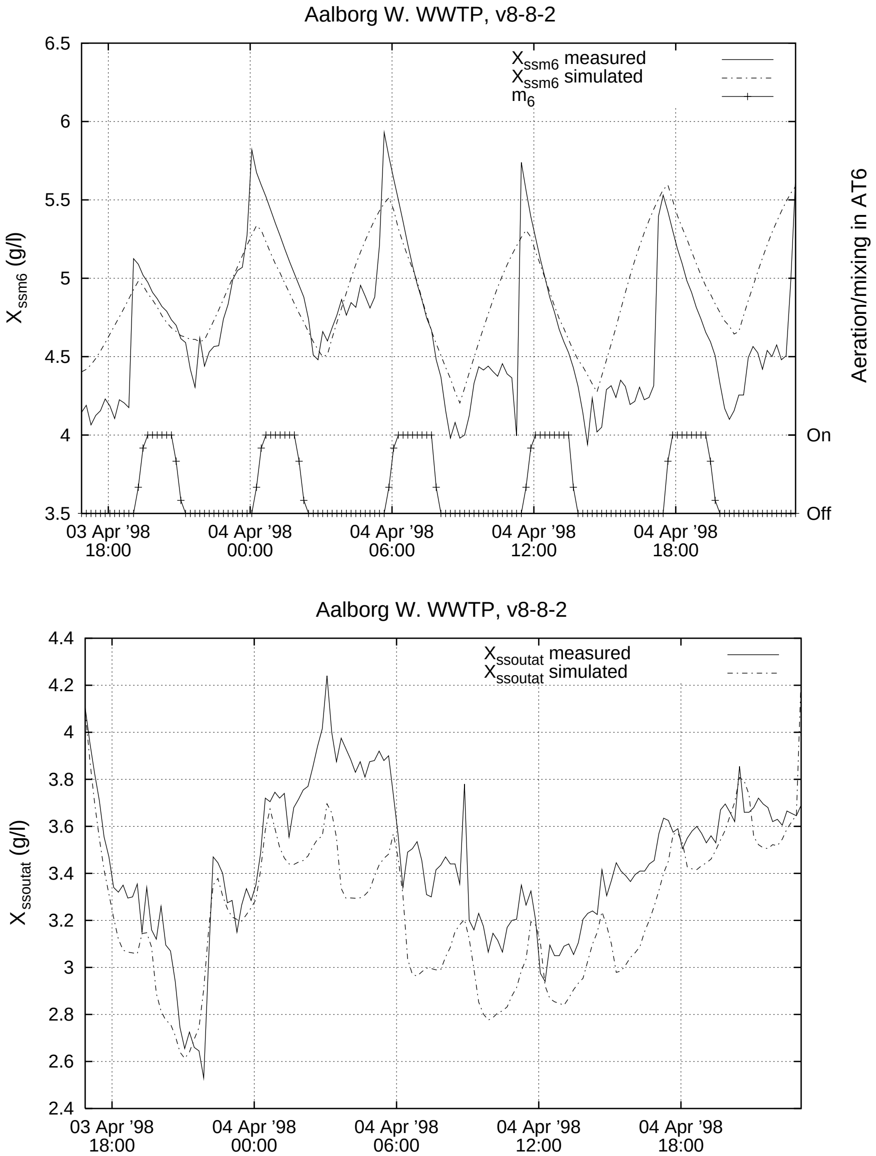

Fig. 5. Measured and simulated SS concentrations in aeration tank 6 and in the effluent from the aeration tanks, validation data set.

best result was obtained with the parameter values for the estimation data set, hence, these values were used to make the graphs. The fact that the values of V_0 and n_v for the estimation data set performed better with the validation data set indicates that the sludge settling model, the suction depth model and the $X_{ssoutat}$ model are interdependent. These should thus be regarded as a single sub-model, and not independent sub-models. However, the estimates of the suction depth model parameters (d_0 and b_{suct}) are considered realistic, as they result in suction depths from approx. 1.3 m to approx. 1.6 m for $Q_i + Q_r$ between 4000 and 12,000 m^3/h.

The model is found to perform well as regards the SS concentrations in aeration tank 6 for both the estimation data set and the validation data set. The simulated SS concentrations in the effluent from the aeration tanks are not as good for the validation data set as for the estimation data set. This indicates that the combined model consisting of the sludge settling model, the suction depth model and the $X_{ssoutat}$ model could be refined.

The estimation method relies on the assumption that the observation noise is white. The validity of this assumption is checked by use of cumulative residual

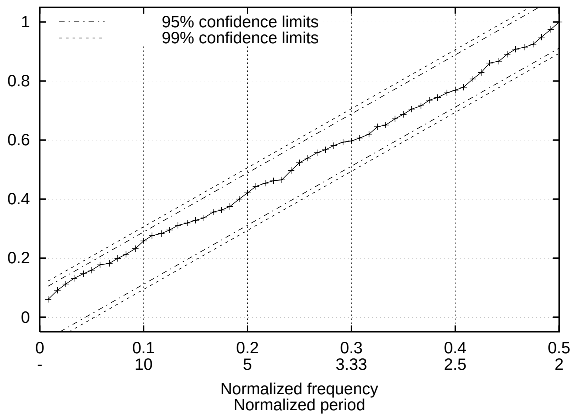

Fig. 6. Cumulative residual periodogram for $X_{ssoutat}$ with 95% and 99% confidence limits, estimation data set.

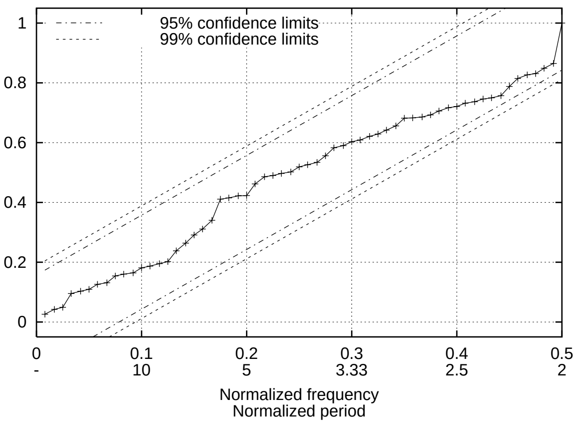

Fig. 7. Cumulative residual periodogram for $X_{ssoutat}$ with 95% and 99% confidence limits, validation data set.

periodograms, see Figs. 6 and 7. As only the observations of X_{ss6} in aerated periods are reliable, the non-aerated periods result in residuals that cannot be considered to be generated by a white noise process. Hence, the cumulative residual periodograms are shown only for the $X_{ssoutat}$ observations. The confidence limits for the periodograms are calculated using the Kolmogorov–Smirnov test principle [15,16]. As the periodograms are between the confidence limits, the $X_{ssoutat}$ residuals can be considered to be white noise. Note that the confidence band for the validation data is wider than for the estimation data. This is caused by the fact that the estimation data set contains more observations than the validation data set.

6. Conclusions

A model for the SS concentrations in and out of the aeration tanks in an alternating WWTP is proposed, and the parameters are estimated using the maximum likelihood method.

The estimated model shows good agreement between simulated and measured SS concentrations in the aeration tanks and in the effluent from these at Aalborg West WWTP. However, improvements are still possible. The sub-models for the sludge settling velocity, the suction depth and the SS concentration out of the aeration tanks are subjects for refinements, and the model should be tested under conditions with more flow variation as well as at other WWTPs. Furthermore, the inclusion of the secondary clarifiers in the model is an important improvement, as the objective is to keep the effluent to the receiving waters to a minimum.

Due to time delays in the aeration tanks the model simulations are 0.8 h ahead of the measurements. Combined with an influent flow forecast horizon of approximately 1 h, the SS concentrations out of the aeration tanks can be predicted almost 2 h ahead. This horizon is considered to be sufficient for selecting the optimal control action.

The proposed model is a valuable tool for designing control algorithms for ATS. By applying the models, it is possible to forecast the SS concentration in the effluent from the aeration tanks. The predictions can be used to choose the best control action, i.e. whether to change the flow direction and switch aeration and mixing on or off, within the limitations, caused by the nutrient removal processes.

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References

[1] Nielsen M, Önnerth T. Improvement of a recirculating plant by introducing STAR control. Water Sci Technol 1995;31(2):171–80.

[2] Önnerth T, Bechmann H. Implementation of and results from advanced on-line computer control of a BNR recirculating plant. New and Emerging Environmental Technologies and Products Conference for Wastewater Treatment and Stormwater Collection, volume 11, Water Environment Federation, 1995. p. 51–72.

[3] Nielsen MK, Bechmann H, Henze M. Modelling and test of aeration tank settling ATS. Water Sci Technol 2000;41(9):179–84.

[4] Vesilind PA. Design of prototype thickeners from batch settling tests. Water Sewage Works 1968;115:302–7.
[5] Vesilind PA. Treatment and disposal of wastewater sludge, 1st ed. Ann Arbor, MI: Ann Arbor Science, 1979.
[6] Grijspeerdt K, Vanrolleghem P, Verstraete W. Selection of one-dimensional sedimentation: models for on-line use. Water Sci Technol 1995;31(2):193–204.
[7] Dupont R, Dahl C. A one-dimensional model for a secondary settling tank including density current and short-circuiting. Water Sci Technol 1995;31(2):215–24.
[8] Ekama GA, Barnard JL, Günthert FW, Krebs P, McCorquodale JA, Parker DS, Wahlberg EJ. Secondary settling tanks: theory, modelling, design and operation. IAWQ Scientific and Technical Report No. 6, IAWQ, London, 1997.
[9] Härtel L, Pöpel HJ. A dynamic secondary clarifier model including processes of sludge thickening. Water Sci Technol 1992;25(6):267–84.
[10] Øksendal B. Stochastic differential equations, 4th ed. Berlin: Springer, 1995.
[11] Kloeden P, Platen E. Numerical solutions of stochastic differential equations. 2nd ed. Berlin: Springer, 1995.
[12] Bechmann H, Madsen H, Poulsen NK, Nielsen MK. Grey box modelling of first flush and incoming wastewater at a wastewater treatment plant. Environmetrics 2000;11(1): 1–12.
[13] Madsen H, Melgaard H. The mathematical and numerical methods used in CTLSM—a program for ML-estimation in stochastic, continuous time dynamical models. Technical Report No. 7/1991, Institute of Mathematical Statistics and Operations Research, Technical University of Denmark, Lyngby, Denmark, 1991.
[14] Melgaard H, Madsen H. CTLSM continuous time linear stochastic modelling. in: Bloem J, editor. Workshop on application of system identification in energy savings in buildings. Ispra, Italy: Institute for Systems Engineering and Informatics, Joint Research Centre, 1993. p. 41–60.
[15] Kendall MG, Stuart A. The advanced theory of statistics, vol. 2, 4th ed. London: Griffin, 1979.
[16] Box GEP, Jenkins JM. Time series analysis: forecasting and control. San Francisco, CA: Holden-Day, 1976.