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Modelling the heat consumption in district heating systems using a grey-box approach

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Abstract

The heat consumption in a large geographical area is considered together with climate measurements on a single location in the area. The purpose is to identify a model linking the heat consumption to climate and calendar information.

The process of building a model is split into a theoretical based identification of an overall model structure followed by data-based modelling, whereby the details of the model are identified. This approach is sometimes called grey-box modelling, but the specific approach used here does not require states to be specified. Overall, the paper demonstrates the power of the grey-box approach.

Keywords: District heating; Consumption; Climate; Modelling; Grey-box

1. Introduction

This paper deals with grey-box modelling of the heat consumption in a large district heating system. The grey-box approach for modelling combines physical knowledge with data-based (statistical) modelling; physical knowledge provides the main structure and statistical modelling provides details on structure and the actual coefficients/estimates. This is advantageous since the physical knowledge reduces the model-space which must be searched, whereby the validity of the statistical methods is better preserved, i.e. the grey-box approach helps preventing overfitting.

The specific analysis has been carried out as part of a larger project described in [21]. The purpose of this project was to develop methods for on-line prediction of the heat consumption based on meteorological forecasts and information from the SCADA system. It was decided that the prediction methods must be easy to apply in practice, and therefore, they should be able to adapt to slow changes of the system. In this paper, however, we do not directly consider prediction.

Previously grey-box modelling has been used in building modelling, but often for very well-defined systems and with the purpose of estimating thermal characteristics of the building components. Madsen and Holst [15] models a test building using stochastic differential equations with two states: Temperatures of indoor air and heat accumulating medium, respectively. Using measurements, the heat capacities and resistances are estimated. The same type of model is used by Madsen and Melgaard [16]. Norlén [22] models a homogeneous slab and considers methods for estimation of the thermal resistance and capacitance. With the purpose of obtaining reliable estimates of thermal capacities and resistances, Hammarsten [7] considers methods for approximation of partial differential equations using lumped parameter models. Arvastson [2] considers modelling and operation of a district heating system, and here, the focus is not on estimation of thermal characteristics. The heat dynamics of the buildings is modelled as a second-order system originating from a simple generic building model. However, the model does not include solar radiation and ventilation/infiltration. Jonsson et al. [12] use ARX-models to estimate the hot tap water consumption by splitting the total consumption in a part dependent upon climate variables and a part not dependent on climate variables. Schuricht and Tauscher [23] discuss the influence

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of climate variables on the heat load and propose a model for the daily load. Madsen et al. [17] use an almost purely data driven (black-box) method to obtain a model for the heat load in a district heating system. A similar approach is used by Wiklund [24]. The approach presented in this paper is not focused on estimation of thermal characteristics of building elements but on the thermal characteristics of an entire district heating system. Input–output relations are used, and therefore, the problem of defining state variables is avoided. In the paper, it is argued that since only measurements of heat consumption and climate variables on a hourly basis are available this limits the identifiability of the model, e.g. some time constants must be completely excluded from the model.

The structure of the paper is as follows. In Section 2, the data and preprocessing of these are described. Using simple stationary relations and approximations of the dynamics, a basic model structure is derived in Section 3. Using the basic model structure as a guidance, the details of the model is determined in Section 4 using a stepwise approach. Finally, in Section 5, we conclude on the paper.

2. Data

The data consists of hourly measurements of heat consumption and climate for the period from July 1995 to June 1996. Furthermore, data on the types of the individual days are used.

2.1. Measurements

The measurements of heat consumption are supplied by Vestegnens Kraft-varmeselskab I/S (VEKS), Denmark and consist of hourly measurements of the heat supplied from the VEKS transmission system to the local distributors. The unit of this measurement is ‘GJ’ and since the measurement is related to the past hour the unit GJ/h will be used in this report. The total annual consumption is more than 8000 TJ.

The climate measurements consist of recordings of the air temperature in °C, wind speed in m/s and global radiation in W/m^2 as averages over the past 10 min up to the full hour. The measurements are performed at Højbakkegård in Taastrup (near Copenhagen), Denmark by the Department of Agricultural Sciences, The Royal Veterinary and Agricultural University, Denmark [11].

2.2. Calendar information

With the purpose of taking into account the different pattern of consumption for different types of days these are grouped into ‘‘working’’, ‘‘half-holy’’ and ‘‘holy’’ days. Half-holy days include Saturdays. These data have been supplied by Elkraft System, Ballerup, Denmark.

2.3. Outliers

The data are thoroughly checked with respect to outliers, see [21, Sec. 2.1]. For the measurements of heat consumption, 56 recordings are missing. Furthermore, 354 recordings are identified as outliers and hence treated as missing values. Overall, nearly 5% of the data are missing (see also Fig. 1). To avoid imposing uncontrolled assumptions on the modelling process, it is not attempted to replace the data missing for the heat consumption. For the climate measurements, the fractions of outliers are relatively low. Since, filtering of these climate variables is not possible in case of missing values, the outlying and missing climate variables are replaced by appropriate values using methods like: (i) robust non-parametric regression on air temperature measurements performed by VEKS and (ii) time-series decomposition and robust estimation and smoothing. For details, see [21, Sec. 2.2].

2.4. Solar radiation

The measurement of global radiation is the solar radiation on a horizontal plane. For predicting the heat consumption, the solar radiation hitting the walls of the houses is presumably more adequate. Here, a square pillar facing the four quarters of the globe will be considered. For each of the four walls, the solar radiation per square meter is calculated

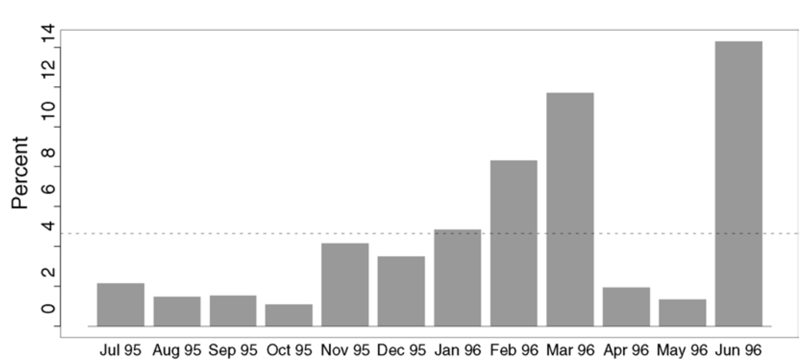

Fig. 1. Percentage of missing values of data on heat consumption by month. Outliers are treated as missing values. The dotted line indicates the overall fraction of missing values.

as the sum of the direct, diffuse and reflected contributions. The average over the four walls is used as a measure of the contribution of the solar radiation to the heating of the buildings. For details, see [21, Sec. 2.3].

3. Model structure

In this section, physical and behavioral characteristics of the system are used to select an overall model structure for use during the statistical modelling process. First, the stationary heat transfer through a wall, the stationary heat and radiation transfer through a window and heat loss due to ventilation are described. A simple model for the dynamic response on climate is postulated and special considerations related to the heat consumption of a larger group of buildings are described. In all of the above aspects actual values of physical constants, e.g. thermal conductivity, are not used. Instead, the structure of the deduced model is used to obtain a structure of an appropriate statistical model.

3.1. Stationary physical relations

In this section, the stationary heat transfer through a wall and a window is described. For further reading, see, e.g. [10]. More details can be found in [21].

3.1.1. Heat transfer through a wall

Denote the indoor air temperature by T_i, the ambient air temperature by T_a, the solar radiation orthogonal to the wall by R_0 and the heat flux by $\dot{Q}$ (positive when heat is transported from the inside to the outside of the building).

Taking into account heat transfer, convection and solar radiation it is simple to derive stationary relations for the two boundary layers and for the wall. Hereafter, simple arithmetics lead to the following stationary relation:

$$\dot{Q} = U(T_i - T_a) - U\frac{\epsilon}{h_o}R_0, \tag{1}$$

where $U = (1/U_w + 1/h_i + 1/h_o)^{-1}$. Here, ϵ is the fraction of R_0 not reflected, U_w is the thermal conductivity of the wall divided by the wall thickness and h_i and h_o are the convection heat coefficients on the inside and outside of the wall, respectively. When the wall consists of layers of different materials, U_w can be found using the thermal conductivity and thickness of each layer [10]. It is assumed that h_i will be nearly constant due to the fairly constant indoor environment, while it is expected that h_o will be influenced by the wind speed.

3.1.2. Energy transfer through a window

The energy transfer through a window consists of conduction/convection as described above and solar radiation directly transmitted through the window. Here, we assume that all solar radiation not reflected is transferred through the window as radiation. Consequently, the following equation is obtained for the energy flux through the window:

$$\dot{Q} = -\epsilon R_0 + U(T_i - T_a), \tag{2}$$

where $U = (1/U_{win} + 1/h_i + 1/h_o)^{-1}$, here, U_{win} is the thermal conductivity divided by the window thickness. The equation is valid also, when the window consists of multiple layers of glass separated by gas or atmospheric air.

3.1.3. Ventilation

The warm air inside the buildings is gradually replaced by cold air from the surroundings. Here, we call this process ventilation, although it is often called infiltration in the technical literature. The heat flux needed to heat the air from the ambient air temperature T_a to the indoor temperature T_i can be expressed as:

$$\dot{Q} = \mathcal{C}\dot{V}(T_i - T_a), \tag{3}$$

where $\dot{V}$ is the flow of air through the building and $\mathcal{C}$ is the product of the specific heat capacity of the air and the mass density of the air. It is evident that for some buildings $\dot{V}$ depends on the wind speed. Also, the humidity of the air might influence $\mathcal{C}$. Since the amount of water vapour corresponding to a specific relative humidity is strongly dependent on the temperature, it is plausible that the variation in $\mathcal{C}$ to some extent can be explained by variation of the ambient air temperature.

3.2. Approximate dynamics

Here, the stationary relations described in Section 3.1 will be modified to take into account the dynamic response on changing climate conditions. However, the actual building dynamics will not be entirely modelled. This seems reasonable since for instance a constant indoor temperature will effectually eliminate the heat storage capacity of the floor and internal walls. Filtering of climate variables is considered in this section. As described in, e.g. [25], when a linear dynamic system is sampled, and if the input can be considered constant within the sampling interval, the input–output relation of the discrete time system can be described by rational transfer functions $H(q) = B(q^{-1})/A(q^{-1})$, where A and B are polynomials and q^{-1} is the backward shift operator ($q^{-1}x_t = x_{t-1}$). For this reason, it seems reasonable to use rational transfer functions when filtering the climate variables. Furthermore, it will be required that the stationary gain of the filters, $H(1)$, is one. Finally, the dot above Q will be dropped since it is only a matter of a constant if we consider the average heat flux from $t-1$ to t, or the heat consumption over the time interval $t-1$ to t.

The heat transfer related to a wall consists of convection in the boundary layers and conduction through the wall. It is assumed that when the sampling period is 1 h, as in this study, the dynamics of the boundary layers can be neglected. Furthermore, it is clear that the ambient air temperature T_a, wind speed W, and solar radiation orthogonal to the wall R_0 must be low-pass filtered. Also, since this filter is related to the materials of the wall the same filter can be used for all three variables. Furthermore, it is reasonable to assume that the dynamic of the walls behaves such that the transfer function $H_1(q)$ will have real and positive poles only. Assuming a fairly constant in-door temperature results in the following equation describing the approximate dynamics of a wall:

$$Q_{1,t} = U_1(H_1(q)W_t)[T_{i,t} - H_1(q)T_{a,t}] - \epsilon_1 \frac{U_1(H_1(q)W_t)}{h_{01}(H_1(q)W_t)} H_1(q) R_{0,t}^{\text{wall}}, \quad (4)$$

where $Q_{1,t}$ is the energy transferred through the wall during the hour starting at $t-1$ and ending at t, $T_{i,t}$ the (unknown) indoor temperature, $T_{a,t}$ the ambient air temperature, W_t the wind speed and $R_{0,t}^{\text{wall}}$ is the solar radiation orthogonal to the wall.

For the energy transfer through a window, it is evident that the dynamics consists of the dynamics of the glass and air in the window and on the dynamics of indoor building elements (floor, indoor walls, etc.), which heats the air after being heated by the solar radiation. Consequently, two transfer functions are needed to describe the approximate dynamics of the energy transfer through a window. However, since the sampling interval is 1 h, the dynamics related to the glass of the window is neglected. Again, assuming a fairly constant indoor temperature results in the following equation describing the approximate dynamics related to a window:

$$Q_{2,t} = -\epsilon_2 H_2(q) R_{0,t}^{\text{win}} + U_2(W_t)[T_{i,t} - T_{a,t}], \quad (5)$$

where $Q_{2,t}$ is the energy which, due to the windows, is needed during the hour starting at $t-1$ and ending at t to maintain the indoor temperature at $T_{i,t}$, while $R_{0,t}^{\text{win}}$ is the solar radiation orthogonal to the window. The remaining variables are described above. Note that the solar radiation must enter through the window and heat, e.g. the floor before it affects the energy needed to maintain the indoor temperature. For this reason, $Q_{2,t}$ is not the energy transferred through the window during the time period ranging from $t-1$ to t, but the energy low-pass filtered by the floor, indoor walls, etc. It seems plausible that the slowest dynamics are related to $H_1(q)$ followed by $H_2(q)$.

With respect to the heat loss due to ventilation, it will be assumed that the dynamics are negligible, when a sampling interval of 1 h is used. Hence,

$$Q_{3,t} = \mathcal{C}_3(T_{a,t})\dot{V}_3(W_t)[T_{i,t} - T_{a,t}], \quad (6)$$

where $Q_{3,t}$ is the energy which, due to the ventilation, is needed during the hour starting at $t-1$ and ending at t to maintain the indoor temperature at $T_{i,t}$. The remaining variables are described above.

Note that since the climate measurements are averages over the past 10 min before the hour, since these are only measured at one location (Taastrup), and since the heat consumption is related to a large geographical area (Copenhagen, Roskilde, Solrød; approximately 60 km × 30 km), it is possible that, for both (5) and (6), low-pass filtering will be advantageous.

3.3. Behavioral considerations

The heat consumption of an area consists of both heat loss to the surroundings $Q_{L,t} = Q_{1,t} + Q_{2,t} + Q_{3,t}$, "free" heat $Q_{F,t}$ coming from, e.g. electrical equipment but also from humans, and energy needed for hot tap water $Q_{W,t}$. The energy needed for heating $Q_{H,t}$ can be expressed as:

$$Q_{H,t} = [Q_{L,t} - Q_{F,t}]_+. \quad (7)$$

The truncation of negative values, indicated by the subscript (+), is used since district heating cannot be used for cooling. For normal households in Denmark, ventilation is used instead.

Let $\delta_{c,t}$ be the fraction of consumers reacting on the climate, and let $\delta_{p,t}$ be the fraction of the potential consumption active at time t, i.e. $\delta_{p,t}$ accounts for holidays and $\delta_{c,t}$ accounts for the fact that during the summer period, almost no consumers react on the climate and that they do not all start/stop reacting on the climate at the same time of year. If assuming that $\delta_{p,t}$ only affects the consumption of hot tab water and free heat, the total heat consumption Q_t over the hour starting at $t-1$ and ending at t can be expressed as:

$$Q_t = \delta_{c,t}[Q_{L,t} - \delta_{p,t}Q_{F,t}]_+ + \delta_{p,t}Q_{W,t}. \quad (8)$$

3.4. Identifiable model

The quantities of the model defined by (4)–(6) and (8) cannot be estimated with the measurements available, which are the total heat consumption Q_t, the ambient air temperature $T_{a,t}$, the wind speed W_t, and the global radiation $R_{g,t}$. The aim of this section is to reach a model structure containing quantities, which can be estimated from the data. It will be assumed that the truncation in (8) can be neglected since the truncation will only become active a few times during the spring[1] ($\delta_{c,t}$ will be close to zero during the summer period). Therefore, the model for the total heat consumption can be expressed as:

$$Q_t = \delta_{c,t}Q_{L,t} - \delta_{c,t}\delta_{p,t}Q_{F,t} + \delta_{p,t}Q_{W,t}. \quad (9)$$

In the model, the solar radiation on walls and windows are unknown. The approach in this paper is to replace both with the global radiation or with a variable, which can be calculated from the global radiation, solar evaluation, and time of the year (cf. Section 2.4). In the following, this quantity will be denoted R_t. Note that this will distort ϵ_1 and

[1] This is especially true in Denmark due to the coastal climate.

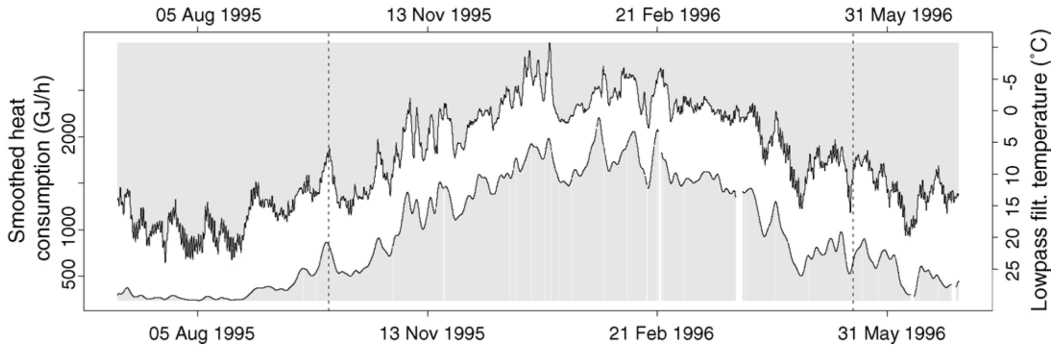

Fig. 2. Cave-plot of low-pass filtered air temperature (top) and smoothed heat consumption (bottom). The vertical lines indicate 10 Oct 1995 00:00 and 15 May 1996 24:00, respectively.

ϵ_2 in (4) and (5), respectively, by a factor. Using (4)–(6) and (9) the total heat consumption can be expressed as:

$$\begin{aligned} Q_t = {} & \delta_{\mathrm{p},t} Q_{\mathrm{W},t} - \delta_{\mathrm{c},t}\delta_{\mathrm{p},t}, Q_{\mathrm{F},t} - \delta_{\mathrm{c},t}\epsilon_2 H_2(q) R_t \\ & + \delta_{\mathrm{c},t} U_1(H_1(q)W_t) T_{\mathrm{i},t} - \delta_{\mathrm{c},t} U_1(H_1(q)W_t) H_1(q) T_{\mathrm{a},t} \\ & - \delta_{\mathrm{c},t}\epsilon_1 \frac{U_1(H_1(q)W_t)}{h_{01}(H_1(q)W_t)} H_1(q) R_t \\ & + \delta_{\mathrm{c},t} U_2(W_t) T_{\mathrm{i},t} - \delta_{\mathrm{c},t} U_2(W_t) T_{\mathrm{a},t} \\ & + \delta_{\mathrm{c},t} \mathcal{C}_3(T_{\mathrm{a},t}) \dot{V}_3(W_t) T_{\mathrm{i},t} - \delta_{\mathrm{c},t} \mathcal{C}_3(T_{\mathrm{a},t}) \dot{V}_3(W_t) T_{\mathrm{a},t}. \end{aligned} \quad (10)$$

If the dependence of $\mathcal{C}_3$ on $T_{\mathrm{a},t}$ is neglected, and if it is assumed that the heat needed for hot tap water $Q_{\mathrm{W},t}$ and the "free" heat $Q_{\mathrm{F},t}$ depend on the time of day h_t^{24} and on the type of day Υ_t, the following model follows from (10):

$$\begin{aligned} Q_t = {} & \delta_{\mathrm{h},t} \mu(h_t^{24}, \Upsilon_t) + \delta_{\mathrm{c},t} a_{20} H_2(q) R_t \\ & + \delta_{\mathrm{c},t} a_{11}(H_1(q)W_t) + \delta_{\mathrm{c},t} a_{12}(H_1(q)W_t) H_1(q) T_{\mathrm{a},t} \\ & + \delta_{\mathrm{c},t} a_{10}(H_1(q)W_t) H_1(q) R_t \\ & + \delta_{\mathrm{c},t} a_{21}(W_t) + \delta_{\mathrm{c},t} a_{22}(W_t) T_{\mathrm{a},t}. \end{aligned} \quad (11)$$

In (11), $\delta_{\mathrm{h},t}$ is used instead of $\delta_{\mathrm{c},t}$ and $\delta_{\mathrm{p},t}$ in $\delta_{\mathrm{p},t} Q_{\mathrm{W},t} - \delta_{\mathrm{c},t}\delta_{\mathrm{p},t} Q_{\mathrm{F},t}$. If $\mu(\cdot,\cdot)$ is modelled by a linear model and if the parameters of the transfer functions $H_1(q)$ and $H_2(q)$ are known, then (11) is a varying-coefficient model as described by Hastie and Tibshirani [8]. It is noted that the coefficient-functions are time-varying.

4. Modelling using data

The initial investigation of the model (11) will be complicated if the variation over time is to be taken into account. For this reason, a winter period, in which it is reasonable to neglect the time-variation, is selected. Time-adaptive estimation in the resulting model is proposed for handling the time-variation, see [21, Chap. 6] and Sections 3.3–4 above.

It is notable that even without the time-variation, the model is still somewhat complicated to investigate since there do not seem to be an obvious parametrization for the dependence of the coefficients on the wind speed. If the transfer-functions are known, the model is a varying-coefficient model as described by Hastie and Tibshirani [8], and it can be separated into a linear model and two conditional parametric models, see [21, Sec. 3.1–2] and [4,18]. For this reason, it seems obvious to start the analysis with the part of the model, which is expected to be most important, i.e. the conditional parametric model describing the heat transfer through the wall. Hereafter, the model is extended with the linear model related to diurnal variation and to solar radiation through windows, followed by the conditional parametric model intended for the description of the ventilation, i.e. the last line of (11). Finally, based on an analysis of the model error, the model will be extended further to model the correlation structure of the error.

4.1. Period without time-variation

To a large extent, the heat consumption follows the ambient air temperature. From previous studies [20, Sec. 8.3.2], it is known that the dynamic response on the temperature, for a discrete time model with hourly data, can be well-described by a transfer-function with one pole at approximately 0.93. Here, it is further assumed that the transfer-function has no zeros. Fig. 2 shows a cave-plot [3] of $0.7/(1 - 0.93\,q^{-1}) T_{\mathrm{a},t}$ and a smoothed version of the heat consumption. The smoothed heat consumption is created using a local quadratic approximation in time and a fixed bandwidth of 3 days. Using this smoother, noise and diurnal variation are removed without introducing a phase-shift. The cave-plot is constructed so that if the dependence of the smoothed heat consumption on the filtered air temperature is linear and time-invariant, then the vertical distance between the curves is constant.[2]

From the plot, it is seen that during the summer period, there is very limited response on the temperature, but at some stage during the autumn, the heat consumption starts to follow the ambient air temperature. Except for a few occasions, the difference between the curves are approximately constant during the period ranging from 10 October

[2] More details can be found at http://www.imm.dtu.dk/~han/pub/cave-plot.

1995 to 15 May 1996, indicating that in this period, the time-variation can be neglected. In [21, Sec. 5.1], further analyses confirm this observation.

4.2. Structural model

As outlined in the beginning of this section, both the transfer functions, coefficients, and coefficient-functions of (11) must be structurally identified and estimated. This is a complicated task, which we approach by a stepwise modelling procedure guided by the overall structure (11).

Initially, the part of the model related to the heat transfer through walls is considered, i.e. the model:

$$Q_t = a_{11}(H_1(q)W_t) + a_{12}(H_1(q)W_t)H_1(q)T_{a,t} + a_{10}(H_1(q)W_t)H_1(q)R_t + e_t, \quad (12)$$

is used. Based on previous work [20, Sec. 8.3.2], the transfer function is selected as $H_1(q) = (1-\phi)/(1-\phi q^{-1})$ with $\hat{\phi} = 0.93$. With this restriction (12) becomes a conditional parametric model with coefficient-functions $a_{1\cdot}(\cdot)$, which we assume to be smooth. The coefficient-functions are then estimated using local regression [4,5,19], where 5- and 100-fold cross validation are used to select an appropriate bandwidth. After fixing this bandwidth the estimate of the pole ϕ of $H_1(q)$ is fine-tuned, but this results in only a marginal change of $\hat{\phi}$ to 0.94.

With the bandwidth and pole selected as described above the model is extended by adding: (i) the term relating to the solar radiation heating the interior of buildings and (ii) the diurnal variation modelled as a free parameter for each hour of the day with different parameters for working days, half-holy days, and holy days. Consequently, the model becomes:

$$Q_t = \mu(h_t^{24}, \Upsilon_t) + a_{20}H_2(q)R_t + a_{11}(H_1(q)W_t) + a_{12}(H_1(q)W_t)H_1(q)T_{a,t} + a_{10}(H_1(q)W_t)H_1(q)R_t + e_t, \quad (13)$$

where the quantities are defined in Section 3. The transfer function $H_2(q)$ is approximated by a finite impulse response (FIR) filter, and the parameter a_{20} is included in the filter:

$$a_{20}H_2(q) = \sum_{i=0}^{24} h_{2,i}\, q^{-i}. \quad (14)$$

In this way, the model becomes semi-parametric, the linear part being $\mu(h_t^{24}, \Upsilon_t) + a_{20}H_2(q)R_t$. A maximum lag of 24 is chosen based on [20, Sec. 8.3.3]. Given the bandwidth and $H_1(q)$ the coefficients in (14) and $\mu(h_t^{24}, \Upsilon_t)$ and the coefficient-functions can be estimated using backfitting. However, here, the non-iterative solution described by Hastie and Tibshirani [9, p. 118] is used. The ambiguity of the model, originating from the fact that a constant can be moved between $a_{11}(H_1(q)W_t)$ and $\mu(h_t^{24}, \Upsilon_t)$, is solved by centering the fitted values of the conditional parametric part of the model [9, p.115].

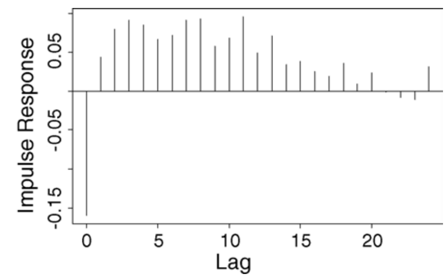

Fig. 3. Estimated impulse response of $a_{20}H_2(q)$ based on the model consisting of (13) and (14).

The resulting estimate of the impulse response is depicted in Fig. 3. The shape of the impulse response indicates that it will be well-approximated by:

$$a_{20}H_{20}(q) = \frac{b_{200} + b_{201}q^{-1} + b_{202}q^{-2}}{1 + a_{201}q^{-1} + a_{202}q^{-2}}. \quad (15)$$

In [21], more detailed investigations indicate that the coefficient-functions $a_{1\cdot}(\cdot)$ are well-approximated by linear functions. If this approximation is used, the full model consisting of (13) and (15) estimated using a least squares criterion corresponds to one optimization problem, with initial values selected as described in [21, p. 54]. The root mean squared error (RMSE) of the residuals of this fit is 92.12 GJ/h compared to 91.53 GJ/h for the semi-parametric model. The increase of 0.53 GJ/h cannot justify the additional degrees of freedom required by the FIR-filter and the coefficient-functions.

Using the model obtained at this stage, the part of the model related to ventilation is added and identified using an approach similar to the approach used for the solar radiation. For details, the reader is referred to [21, Sec. 5.4]. The investigation results in the following model:

$$\begin{aligned} Q_t = {} & \mu(h_t^{24}, \Upsilon_t) + a_{20}H_2(q)R_t + a_{111}H_1(q)W_t \\ & + a_{120}H_1(q)T_{a,t} + a_{121}H_1(q)W_tH_1(q)T_{a,t} \\ & + a_{100}H_1(q)R_t + a_{101}H_1(q)W_tH_1(q)R_t + a_{211,0}W_t \\ & + a_{211,1}W_{t-1} + a_{220,0}T_{a,t} + a_{220,1}T_{a,t-1} + a_{2,00}W_tT_{a,t} \\ & + a_{2,01}W_tT_{a,t-1} + a_{2,10}W_{t-1}T_{a,t} + a_{2,11}W_{t-1}T_{a,t-1} + e_t. \end{aligned} \quad (16)$$

Assuming the transfer function to be known, the model is a linear regression model. The smooth coefficient-functions $a_{1\cdot}(\cdot)$ in (13) are replaced by linear functions, i.e. $a_{1\cdot}(x) = a_{1\cdot 0} + a_{1\cdot 1}x$. To ensure uniqueness of the estimates the constant a_{110} originating from $a_{11}(H_1(q)W_t)$ is included in $\mu(h_t^{24}, \Upsilon_t)$.

The model results in a drastic decrease in the RMSE to 81.46 GJ/h. Dropping the interaction terms in the last two lines of (16) results in a RMSE of 81.65 GJ/h. However, in order to retain the physical interpretation of the model, the terms are kept in the model.

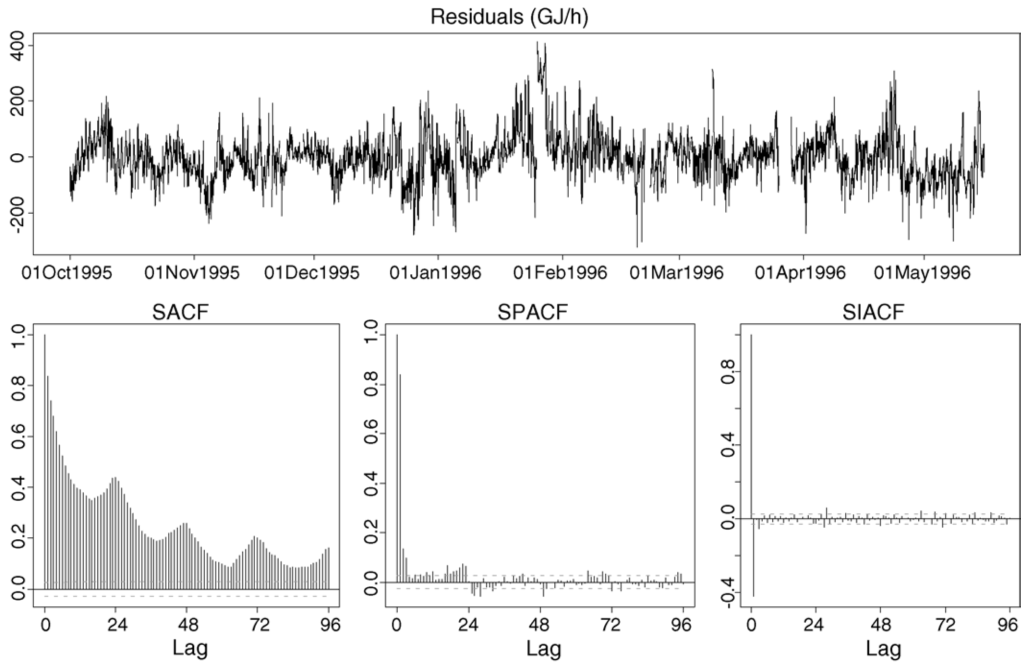

Fig. 4. Residuals from model (16) together with estimated autocorrelation functions of the residuals. The dotted lines mark an approximate 95% confidence interval under the hypothesis of white noise.

4.3. Noise model

Fig. 4 shows the residuals and the sample autocorrelation function (SACF), the sample partial autocorrelation function (SPACF) and the sample inverse autocorrelation function (SIACF) [6,14] and [21, Sec. 3.7], of the residuals from (16). It is noted that the transfer functions are not re-estimated. Iterative ARMA-model building based on the residuals leads to the conclusion that the noise process $\{e_t\}$ in (16) is modelled well by the following autoregressive model:

$$\begin{aligned} e_t &= \frac{1}{A(q^{-1})}\epsilon_t \\ A(q^{-1}) &= 1 - \Sigma_{\ell \in \{1,2,3,23,24\}} a_\ell q^{-\ell}, \end{aligned} \tag{17}$$

where $\{\epsilon_t\}$ is a Gaussian white noise process. The model is identified by reducing a model inspired by a seasonal (24 h) multiplicative structure [21, Sec. 5.5].

The SACF of the residuals, i.e. of estimates of ϵ_t, after re-fitting the model with respect to both the linear and the autoregressive coefficients, but with fixed transfer functions, is displayed in Fig. 5. The result is quite satisfactory. However, the time series plot of the residuals shown in Fig. 6 indicates that the variance is not constant. This aspect is not pursued any further since the purpose of the modelling process is to find a model structure, which can be used for adaptive estimation and prediction.

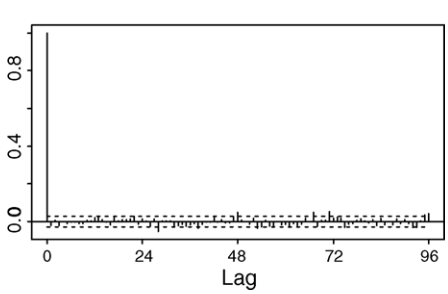

Fig. 5. SACF of residuals (ϵ_t) when using (16) together with (17). The dotted lines mark an approximate 95% confidence interval under the hypothesis of white noise.

The addition of the noise model reduces the RMSE of the in-sample one-step predictions to 38.46 GJ/h, which is less than half the value obtained for (16). However, assuming the climate to be known (16) performs equally well for all horizons. This is not true when including a noise model and in this case the in-sample long-term prediction has RMSE equal to 168.8 GJ/h [21, Sec. 5.7].

4.4. Cross correlations

As indicated by the SACF in Fig. 5, the residuals are uncorrelated. In order to identify possible problems related to the transfer functions the sample cross correlation function (SCCF) between the residuals and filtered input

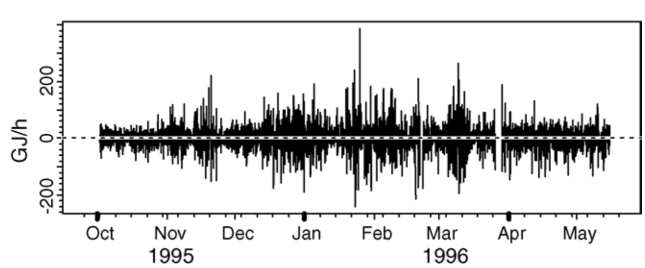

Fig. 6. Residuals (ϵ_t) when using (16) together with (17).

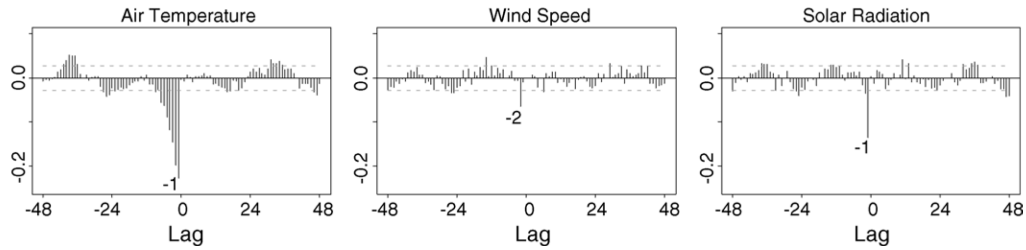

Fig. 7. SCCF of the residuals (ϵ_t) when using (16) together with (17) and the three filtered climate variables. The dotted lines mark an approximate 95% confidence interval under the hypothesis of uncorrelated series.

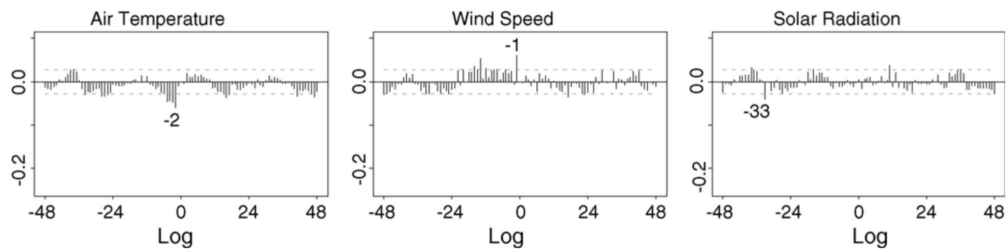

Fig. 8. SCCF of the residuals (ϵ_t) when adding terms containing $T_{a,t+1}$ and R_{t+1} to the model consisting of (16) and (17). The dotted lines mark an approximate 95% confidence interval under the hypothesis of uncorrelated series.

series is calculated [14]. In order to facilitate the interpretation of the estimated cross correlations, the input series are filtered using the convolution filter defined by $A(q^{-1})$ with the coefficients as described above, see also [21, p. 63–64]. The results are shown in Fig. 7.

In the figure, the lags with the maximum absolute correlation are indicated. For the wind speed, this maximum occurs at lag -2 and the value is considered small. Therefore, the wind speed will not be considered further at this place. For the ambient air temperature and the solar radiation, the maximum absolute correlation occurs at lag -1.

For both the ambient air temperature and the solar radiation, the decline of SCCFs towards zero for decreasing lags starting at lag -1, seems to be of first order. This indicates that terms containing $T_{a,t+1}$ and R_{t+1} should be added to the structural part of the model. For this model, the SCCF between the residuals and the filtered input is depicted in Fig. 8. Comparing with Fig. 7, it is seen that most of the cross correlation is removed.

It is seen that future values of the ambient air temperature and solar radiation seem to affect the heat consumption. This is probably due to the geographically distributed consumption and the fact that the climate variables only are measured at a single location. Another explanation could be that the consumers (local network operators) use some prediction device combined with a heat storage facility.

4.5. Likelihood ratio tests

Likelihood ratio tests [13, p. 32] in the model consisting of (16) and (17) are performed. The results of these tests are listed in Table 1. From the tests, it is concluded that the short-term interaction between wind speed and ambient air temperature can be excluded from the model. This is consistent with the root mean square of the residuals from the regression model mentioned at the end of Section 4.2. The test for difference between half-holy and holy days is marginally insignificant at the 5% level. However, by use of appropriate approximations [21, Sec. 5.9], the diurnal variation can be described by use of fewer degrees of freedom per day group. This could easily make the aforementioned test insignificant ($\chi^2(23) = 35.172$), and therefore, the difference between half-holy and holy days is retained in the model. All the remaining tests are significant at the 5% level. Akaike's information criterion[1] indicates that the model where $a_{2,ij} = 0$; $i = 0, 1$; $j = 0, 1$ should be used, i.e. (as expected), it is consistent with the likelihood ratio test.

Table 1
Likelihood ratio tests of selected null hypotheses; in all cases with the model consisting of (16) and (17) as the alternative hypothesis

Excluded	LR statistic	d.f.	p-Value
Short term interaction between wind and temperature[a]	2.613	4	0.624
Short term wind speed[b]	13.794	6	0.032
Short term temperature[c]	247.053	6	0.000
Wind speed	127.451	9	0.000
Solar radiation	271.913	3	0.000
Temperature	435.288	8	0.000
Difference between half-holy and holy days	35.520	24	0.061

[a] $a_{2,00} = a_{2,01} = a_{2,10} = a_{2,11} = 0$.
[b] $a_{2,00} = a_{2,01} = a_{2,10} = a_{2,11} = a_{211,0} = a_{211,1} = 0$.
[c] $a_{2,00} = a_{2,01} = a_{2,10} = a_{2,11} = a_{220,0} = a_{220,1} = 0$.

5. Conclusion

The heat consumption in a large geographic area is considered together with climate measurements on a single location. The purpose of the work described in this paper is to find a model linking the heat consumption to climate and calendar information. Theoretical relations known from the theory of heat transfer have been used to select an initial model structure and data on heat consumption and climate (temperature, wind speed and global radiation) is applied in combination with statistical methods to establish an actual mathematical model of the heat consumption. Such an approach is often called grey-box modelling, but here, the data are more aggregated than is often seen in the literature on grey-box modelling, see, e.g. [15]. The paper demonstrates that even with this kind of aggregation the grey-box approach is powerful.

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