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Models for describing the thermal characteristics of building components

M.J. Jiménez[a,*], H. Madsen[b]

[a]Department of Energy, Energy Efficiency in Buildings Unit, CIEMAT, Av. Complutense 22, E-28040, Madrid, Spain
[b]Informatics and Mathematical Modelling, Technical University of Denmark, Building 321, DK-2800 Lyngby, Denmark

Abstract

Outdoor testing of buildings and building components under real weather conditions provides useful information about their dynamic performance. Such knowledge is needed to properly characterize the heat transfer dynamics and provides useful information for implementing energy saving strategies, for example. For the analysis of these tests, dynamic analysis models and methods are required. However, a wide variety of models and methods exists, and the problem of choosing the most appropriate approach for each particular case is a non-trivial and interdisciplinary task. Knowledge of a large family of these approaches may therefore be very useful for selecting a suitable approach for each particular case.

This paper presents an overview of models that can be applied for modelling the thermal characteristics of buildings and building components using data from outdoor testing. The choice of approach depends on the purpose of the modelling, existence of prior physical knowledge, the data and the available statistical tools. In this paper, a variety of models are outlined and compared, and a strong relationship among a large number of widely used linear and stationary stochastic models is mathematically demonstrated. The characteristics of each type of model are highlighted. Some available software tools for each of the methods described will be mentioned. A case study also demonstrating the difference between linear and nonlinear models is considered.

Keywords: Building energy; Thermal parameters; Outdoor testing; System identification

1. Introduction

The paper firstly outlines the difference between linear and nonlinear models. It considers the strong relationships that exist between linear and time-invariant models. Specifically, the paper describes and compares the following linear models:

- The linear stochastic state space models.
- Stochastic transfer function (Box–Jenkins) models.
- Impulse and step-response functions.
- Frequency response function.
- The linear regression model.

An example, which considers the heat transfer dynamics of a simple test building is given in the later part of the paper. This example also takes nonlinear models into consideration, and in this case it is shown that a nonlinear description of the heat dynamics is necessary.

2. Nonlinear versus linear model building

This section outlines some of the fundamental differences between linear and nonlinear model building. The aim of the modelling effort may be generally expressed as follows.

General model: Find a function h such that $\{\varepsilon_t\}$ defined by

$$h(Y_t, Y_{t-1}, \ldots) = \varepsilon_t \tag{1}$$

is a sequence of independent random variables. Suppose also that the model is *causally invertible*, i.e. the equation above may be 'solved' so that we may write

$$Y_t = h'(\varepsilon_t, \varepsilon_{t-1}, \ldots). \tag{2}$$

*Corresponding author. Tel.: +34 950 387922; fax: +34 950 365015.
E-mail address: Mjose.jimenez@psa.es (M.J. Jiménez).

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Nomenclature

Measured quantities

- P_{aux} energy supplied to the test room by heating and ventilation, W
- G_v global solar radiation on the component surface, W/m^2
- T_i test room air temperature, °C
- T_e outdoor air temperature, °C
- T_{sr} service room air temperature, °C
- T_{ali} test room surface temperature, °C
- T_{ale} exterior surface temperature of the PAS system (Fig. 3), °C

Parameters

- UA thermal transmission coefficient of the tested component, W/K
- gA solar transmittance of the tested component, m^2
- H_{sr} thermal transmission coefficient between test and service rooms, W/K
- H_{ali} thermal transmission coefficient between indoor surface and test room air, W/K
- H_{ale} thermal transmission coefficient between the exterior surface of the PAS system and the test room air, W/K
- C_1 heat capacity of the test room air, J/K
- C_2 effective heat capacity of the indoor test room surface, J/K
- α fraction of solar radiation transmitted through the tested component that reaches the indoor test room surface, dimensionless
- k_{mr2} auxiliary constant coefficient, W/K^4
- k_{mr5} auxiliary constant coefficient, W/K^4
- σ_{ii} system error, J
- e_i measurement error, °C

Suppose that h' behaves sufficiently well so that it can be expanded in a Taylor series:

$$Y_t = \mu + \sum_{k=0}^{\infty} g_k \varepsilon_{t-k} + \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} g_{kl}\varepsilon_{t-k}\varepsilon_{t-l} + \sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty} g_{klm}\varepsilon_{t-k}\varepsilon_{t-l}\varepsilon_{t-m} + \cdots. \tag{3}$$

The functions

$$\mu = h'(0), \quad g_k = \left(\frac{\partial h'}{\partial \varepsilon_{t-k}}\right), \quad g_{kl} = \left(\frac{\partial^2 h'}{\partial \varepsilon_{t-k}\partial \varepsilon_{t-l}}\right) \text{ etc.} \tag{4}$$

are called the *Volterra series* for the process. The sequences $g_k, g_{kl}, \ldots$ are called the *kernels* of the Volterra series.

For *linear systems*:

$$g_{kl} = g_{klm} = g_{klmn} = \cdots = 0. \tag{5}$$

Hence, a linear system is completely characterized by either

$\{g_k\}$: impulse response function

or

$\mathscr{H}(\omega)$: frequency response or transfer function.

In general, there is *no such thing as a transfer function* for a *nonlinear system*. However, an *infinite sequence of generalized transfer functions* may be defined as

$$\mathscr{H}_1(\omega_1) = \sum_{k=0}^{\infty} g_k e^{-i\omega_1 k},$$

$$\mathscr{H}_2(\omega_1,\omega_2) = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} g_{kl} e^{-i(\omega_1 k + \omega_2 l)},$$

$$\mathscr{H}_3(\omega_1,\omega_2,\omega_3) = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty} g_{klm} e^{-i(\omega_1 k + \omega_2 l + \omega_3 m)},$$

$$\vdots$$

Let U_t and Y_t denote the input and output of a nonlinear system, respectively.

For *linear systems* it is well known that:

(L1) If the input is a single harmonic $U_t = A_0 e^{i\omega_0 t}$, then the output is a single harmonic of *the same frequency*, but with the amplitude scaled by $|H(\omega_0)|$ and the phase shifted by $\arg H(\omega_0)$.

(L2) Due to linearity, the *principle of superposition* is valid, and the total output is the sum of the outputs corresponding to the individual frequency components of the input. Hence, the system is completely described if the responses to all frequencies are known, which is what the transfer function supplies.

For *nonlinear systems*, however, neither of the properties (L1) or (L2) hold. More specifically we have that:

(NL1) For an input with frequency ω_0, the output, in general, also contains components with the frequencies $2\omega_0, 3\omega_0, \ldots$ (*frequency multiplication*).

(NL2) For two inputs with frequencies ω_0 and ω_1, the output contains components at frequencies $\omega_0, \omega_1, (\omega_0 + \omega_1)$ and all harmonics of those frequencies (*intermodulation distortion*).

2.1. The nonlinear stochastic state space model

Here, the most general and frequently used nonlinear model in continuous time nonlinear stochastic modelling is briefly mentioned. Due to the possibility of a direct relationship to physics, these types of models are often called grey-box models.

The continuous–discrete stochastic state space model is a model that consists of a set of nonlinear discrete, partially observed stochastic differential equations (SDEs) with measurement noise, i.e.

$$\mathrm{d}x_t = f(x_t, u_t, t, \theta)\,\mathrm{d}t + \sigma(u_t, t, \theta)\,\mathrm{d}\omega_t, \tag{6}$$

$$y_k = h(x_k, u_k, t_k, \theta) + e_k, \tag{7}$$

where $\theta \in \Theta \subset R^p$ is parameter vector; $f(\cdot) \in R^n$, $\sigma(\cdot) \in R^{n\times n}$ and $h(\cdot) \in R^l$ are nonlinear functions; $\{\omega_t\}$ is an n-dimensional standard Wiener process and $\{e_k\}$ is an l-dimensional white noise process with $e_k \in N(0, S(u_k, t_k, \theta))$. An example belonging to this model class is found in Section 8.4.4.

This class of nonlinear models are treated by the tool called continuous time stochastic modelling (CTSM), which is a software for grey-box modelling of physical systems [1]. However, in the next sections attention is limited to linear models.

3. Continuous time linear models in state space form

Firstly, consider the continuous time case, where the stochastic model in state space form is formulated as an extension of an ordinary deterministic lumped model.

A lumped description of dynamic systems is very often proposed, including the heat transfer dynamics of buildings, which frequently are described by a system of linear differential equations. In matrix notation, the equations can be parameterized by *the continuous time deterministic linear state space model*

$$\frac{\mathrm{d}T}{\mathrm{d}t} = AT + BU, \tag{8}$$

where T is the state vector and U is the input vector. The dynamic behaviour of the system is characterized by the matrix A, and matrix B specifies how the input signals (outdoor air temperature, solar radiation, heat supply, etc.) enter the system.

More frequently, however, (8) is not able to predict the future behaviour of the states accurately. To describe the deviation between (8) and the true variation of the states, an additive noise term is introduced. Then the model of the heat transfer dynamics is described by *the continuous time stochastic linear state space model*

$$\mathrm{d}T = AT\,\mathrm{d}t + BU\,\mathrm{d}t + \mathrm{d}w(t), \tag{9}$$

where the mth dimensional stochastic process $w(t)$ is assumed to be a process with independent increments. There are many reasons for introducing such a noise term:

- Deficiencies of the model. For instance the dynamics, as described by the matrix A in (9), might be an approach to the true system.
- Unrecognized inputs. Some variables not taken into consideration may affect the system.
- Measurements of the input are corrupted by noise. In these cases the measured input is regarded as the actual input to the system, and the deviation from the true input is described by $w(t)$.

Eq. (9) describes the transfer of all the states of the system but most likely only some of the states are actually measured. In the linear case it is assumed that only a linear combination of the states is measured, and if T_r is introduced to denote the measured or recorded variables, the following can be written

$$T_\mathrm{r}(t) = CT(t) + e(t), \tag{10}$$

where C is a constant matrix, specifying the linear combination of the states that are actually measured. In practice, however, C frequently acts only as a matrix which picks out the states actually measured.

The term $e(t)$ is the measurement error. The sensors that measure the output signals are subject to noise and drift.

It is often assumed that $e(t)$ is a white noise sequence. Furthermore, $w(t)$ and $e(t)$ are assumed to be mutually independent, which seems to be quite reasonable. However, the measurement error may consist of both a *systematic error* and a *random error* as specified by ISO [2]. In statistical modelling the random error can be accounted for by extending the length of the experiment. The systematic error, on the other hand, is more complicated. Ideally, randomly picked, individually calibrated experiments should be repeated.

This modelling approach has previously been used in [3,4] for the heat transfer dynamics of buildings.

4. Discrete time models in state space form

The finite differences method is often used for transforming differential equations into difference equations. This is, however, very often a crude approximation, and more adequate techniques are preferred (see, for instance, [5]). In the present situation, where the system is assumed to be described by the SDE (9), it is analytically possible to perform an analytical integration, which under some assumptions exactly specifies the system equation in discrete time.

For the continuous time model (9), the corresponding discrete time model is obtained by integrating the differential equation through the sample interval $[t, t+\tau]$. Thus, the sampled version of (9) can be written as

$$T(t+\tau) = \mathrm{e}^{A(t+\tau-t)}T(t) + \int_t^{t+\tau} \mathrm{e}^{A(t+\tau-s)}BU(s)\,\mathrm{d}s + \int_t^{t+\tau} \mathrm{e}^{A(t+\tau-s)}\,\mathrm{d}w(s). \tag{11}$$

Under the assumption that $U(t)$ is constant in the sample interval the sampled version can be written as the following *discrete time model in state space form*:

$$T(t+\tau) = \phi(\tau)T(t) + \Gamma(\tau)U(t) + v(t;\tau), \tag{12}$$

where

$$\phi(\tau) = \mathrm{e}^{A\tau}, \quad \Gamma(\tau) = \int_0^\tau \mathrm{e}^{As}B\,\mathrm{d}s, \tag{13}$$

$$v(t;\tau) = \int_t^{t+\tau} \mathrm{e}^{A(t+\tau-s)}\,\mathrm{d}w(s). \tag{14}$$

If the input is not constant in the sample interval, other methods can be used—see, for instance, [5].

Assuming that $w(t)$ is a Wiener process, $v(t;\tau)$ becomes normally distributed white noise with zero mean and covariance

$$R_1(\tau) = E[v(t;\tau)v(t;\tau)']. \tag{15}$$

The total state space form most frequently includes the measurement equation, which in this case is the same as for continuous time, i.e.

$$T_r(t) = CT(t) + e(t). \tag{16}$$

If the sampling time is constant (equally spaced observations), the stochastic difference equation can be written as

$$T(t+1) = \phi T(t) + \Gamma U(t) + v(t), \tag{17}$$

where the time scale is now transformed in such a way that the sampling time becomes equal to one time unit.

Notice that compared to the continuous time model:

- Equidistant data are assumed and hence the possibility of varying sampling times is lost.
- The direct physical interpretation of the parameters is lost.
- Typically, a larger number of parameters are needed which implies less efficiency and robustness.

5. The transfer function form

The (discrete time) transfer function form is also frequently called the Box–Jenkins transfer function [6]. The transfer function form is introduced by showing how it is obtained from the state space form. Consider the following discrete time state space model:

$$T(t+1) = \phi T(t) + \Gamma U(t) + v(t), \tag{18}$$

$$Y(t) = CT(t) + e(t), \tag{19}$$

where $\{v(t)\}$ and $\{e(t)\}$ are mutually uncorrelated white noise processes with variances R_1 and R_2, respectively.

By using the z-transform the state space form is written as

$$zT(z) = \phi T(z) + \Gamma U(z) + v(z), \tag{20}$$

$$Y(z) = CT(z) + e(z). \tag{21}$$

Eliminating $T(z)$ in (20)–(21) yields

$$Y(z) = C(zI-\phi)^{-1}\Gamma U(z) + C(zI-\phi)^{-1}v(z) + e(z). \tag{22}$$

Note that rational polynomials in z are found before $U(z)$ and $v(z)$. Another possibility, which will be demonstrated later, is to first obtain the innovation form, which is obtained directly from using a Kalman filter on the discrete time model.

If $\{Y_t\}$ is a stationary process (matrix A is stable), then the noise processes in (22) can be concentrated in only one stationary noise process. Following [7] we write

$$Y(z) = C(zI-\phi)^{-1}\Gamma U(z) + [C(zI-\phi)^{-1}K + I]\,\varepsilon(z), \tag{23}$$

or alternatively in *the transfer function form*, *the Box–Jenkins transfer function form* or *the input–output form*:

$$Y(z) = H_1(z)U(z) + H_2(z)\varepsilon(z), \tag{24}$$

where $\{\varepsilon_t\}$ is white noise with variance R, and $H_1(z)$ and $H_2(z)$ are rational functions in z:

$$H_1(z) = C(zI-\phi)^{-1}\Gamma, \tag{25}$$

$$H_2(z) = C(zI-\phi)^{-1}K + I. \tag{26}$$

Matrix K is the stationary Kalman gain. K and R are determined from the values of R_1, R_2, ϕ and C, since

$$K = \phi PC^T(CPC^T + R_2)^{-1}, \tag{27}$$

$$R = CPC^T + R_2, \tag{28}$$

where P is obtained by solving the stationary Ricatti equation

$$P = \phi P\phi^T + R_1 - \phi PC(CPC^T + R_2)CP\phi^T. \tag{29}$$

The ARMAX class of models is obtained in cases where the denominators in (24) for H_1 and H_2 are equal, hence the model is written as

$$A(z)Y(z) = B(z)U(z) + C(z)\varepsilon(z), \tag{30}$$

where A, B and C are polynomials in z.

As shown above, a transfer function can be found from the state space form by simply eliminating the state vector. To go from a transfer function to a state space form is more difficult, since for a given transfer function model, there in fact exists a whole continuum of state space models. The most frequent solution is to choose a canonical state space model—see e.g. [7], or to use some physical knowledge in order to write down a proper connection between desirable state variables, which have to be introduced for the state space form.

Notice that compared to the discrete time state space model it is observed that:

- Decomposition of noise into system and measurement noise is lost.
- The state variable is lost, i.e. the possibility for physical interpretation is further reduced.

6. Impulse and response function models

A non-parametric description of the linear system is obtained from either (24) or (30) by polynomial division followed by an inverse z-transform, i.e.

$$Y(t) = \sum_{i=0}^{\infty} h_i U(t-i) + N(t), \tag{31}$$

where $N(t)$ is a correlated noise sequence. The sequence $\{h_i\}$ is the *impulse response* (*matrix*) *function*.

In the frequency (or z-) domain:

$$Y(z) = H(z)U(z) + N(z), \tag{32}$$

where $H(z)$ is the transfer function, and for $z = e^{i\omega}$ the *frequency response function* (*gain and phase*) is obtained, i.e. $H(\omega) = H(e^{i\omega})$.

Notice that compared to the transfer function models in Section 5 it is now observed that:

- The description of the noise process is lost.
- The non-parametric model hides the number of time constants, etc.

7. The linear regression model

The linear regression model, which describes the stationary situation or steady-state relations, can be obtained directly from the state space models by using the fact that, in the stationary situation, $dT/dt = 0$—or from the state space model in discrete form by $T(t+1) = T(t)$.

Hence, it follows that *the steady-state equation* or *regression model*, which expresses the stationary relationship between influences U and recorded temperature T_r, is given by (from the continuous time model)

$$T_r = -CA^{-1}BU \tag{33}$$

or (from the discrete time model)

$$T_r = C(I - \Phi)^{-1}\Gamma U. \tag{34}$$

Alternatively, the stationary equation is obtained from the (discrete time) transfer function form (24) by making $z = 1$.

Now a description of the dynamics is also lost.

8. Case study

In this section a dynamic test of a building component, carried out in a PASLINK test cell fitted with a pseudo-adiabatic shell (PAS) [8], is analysed in order to illustrate the use of some of the models for describing the heat dynamics of a wall.

Furthermore, both linear and nonlinear models are considered and compared.

The objective of the test is to estimate the UA and gA values of a wall described in Section 8.1.

8.1. Component description

The test component consists of an opaque wall with a double-glazed window which is installed by replacing a removable opaque piece in the centre. The total component surface coincides with the test cell aperture, which in this case is $2.47\,m \times 2.48\,m = 6.13\,m^2$.

The opaque wall is homogeneous and quite simple (see Fig. 1) consisting of a sandwich plywood-insulation structure using the following materials:

- Expanded polystyrene with 200 mm thickness (PS30) and 30 kg/m^3 density in four layers of 50 mm thickness each.
- Phenolic plywood with 12 mm thickness.

Fig. 1. Detail of the opaque part of the test component.

- White melamine veneer on the outer face with 0.7mm thickness.
- Sikaflex 11FC for gluing the layers.
- 16 size M12 nylon bolts, washers and nuts, uniformly distributed, to reinforce construction.

The window, including frame, is $1.25\,m \times 1.50\,m$ with $1.18\,m \times 1.43\,m$ glazing. It is a double-glazed window with plain float glass in a timber frame. The two sheets of glass are fitted in the frame with silicone and secured with timber beading. Desiccant is used to prevent condensation between panes. Fig. 2 shows this component installed in the test cell where it was tested. Further component construction details are found in [9] (Figs. 3 and 4).

8.2. Experimental set-up

The experiment was carried out according to the PASLINK test procedure [10], at the Spanish PASLINK test site at the CIEMAT's Plataforma Solar de Almería (Tabernas, Almería, Spain).

8.3. Data

Two data sets have been considered:

- Series 6: From 4th January 2003 to 15th January 2003.
- Series 7: From 1st February 2003 to 10th February 2003.

A summary of the recorded data is presented in Figs. 5–7. In these data the sampling interval is 1 min, and the averaging and recording interval is 10 min. Both data sets were recorded in winter, but under very different test conditions and hence are useful for validation purposes.

Series 6 corresponds to a test carried out with the test cell facing north, which allows the use of higher range of heating power without overheating. However, the solar radiation input is very weak under these conditions.

Fig. 2. Test component installed in a test cell.

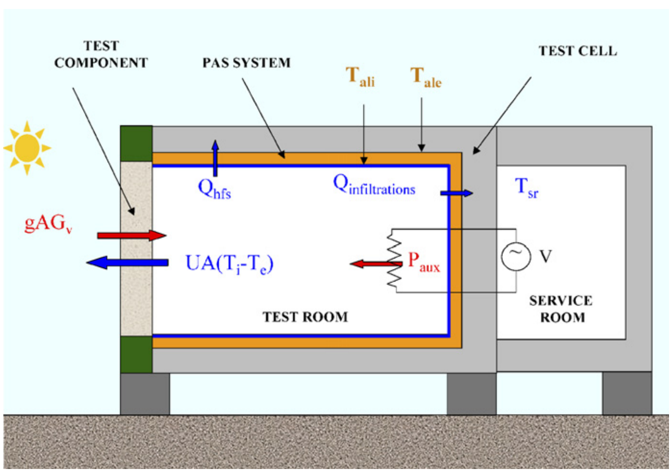

Fig. 3. Diagram of main heat exchange in the test room.

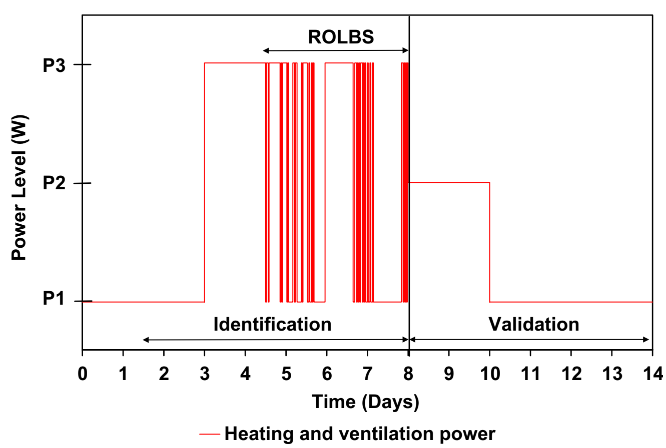

Fig. 4. Typical heating power sequence. P_1 = low power (air circulation fan), P_2 = medium power, P_3 = high power.

Series 7 was recorded with the test cell facing south when the sun was in its lowest position, so the solar radiation incident on the component is very high, and heating power is kept very low to avoid overheating in the test room.

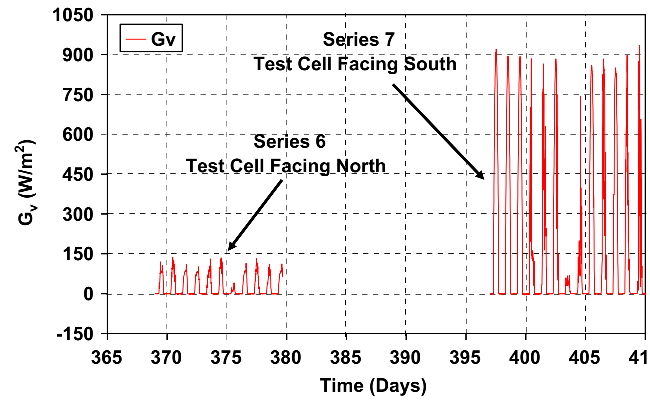

Fig. 5. Global vertical solar radiation.

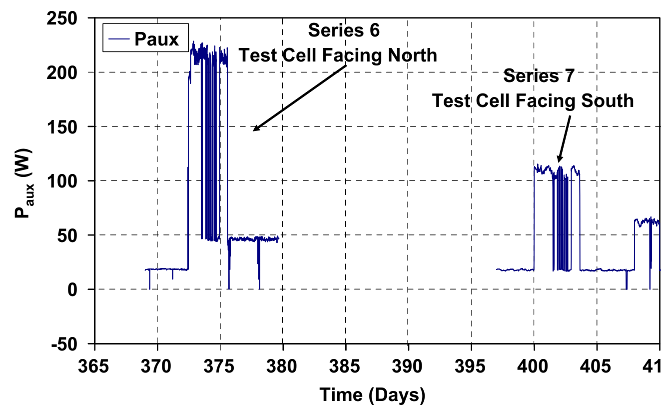

Fig. 6. Heating and ventilation power.

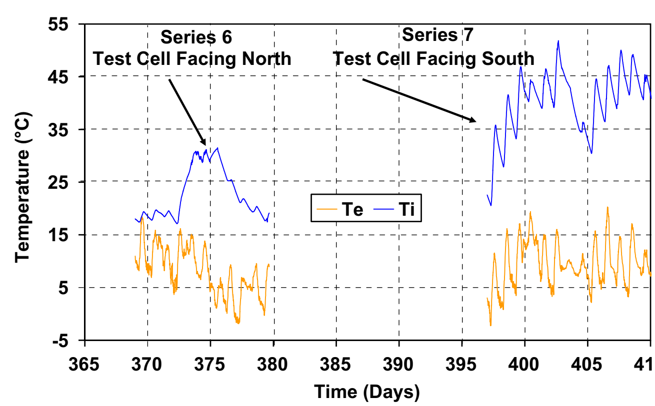

Fig. 7. Indoor and outdoor temperatures.

Series 7 was used for estimation and Series 6 for validation.

Both data sets were obtained setting a heating regime, as shown in Fig. 4, using a randomly ordered logarithmically distributed binary sequence (ROLBS), according to the PASLINK test procedure [10]. The maximum power level should be calculated to ensure that the mean test room temperature difference between the low and high parts should be at least 10 K, but preferably 20 K, without exceeding the safety limits of the test cell. However, due to the heavy insulation in the test cell and in the test building component, the range of the heating sequence was limited. As a result, its effectiveness as a test strategy for exciting the system is limited, and consequently identification of the required parameters becomes more difficult.

8.4. Models

Several models and methods to estimate the required parameters were considered. The main differences between the models are the simplifications related to the different physical effects assumed in each case.

The model selection sequence began by analysing the simplest models, assuming as much simplification as reasonable, and aiming for the simplest model that fits the physical assumptions, while still taking into account the imposed boundary and test conditions.

As these simplified models were insufficient to properly estimate the required parameters, it was necessary to include some of the physical effects initially assumed to be negligible. As a consequence of these non-negligible effects having been included, more mathematically elaborate models and methods are needed.

A summary of the most representative stages of this model selection process is presented in this section. Furthermore, a model representing each stage is described.

Notice that for all models, test component performance is assumed to be linear.

8.4.1. Model 0: linear model in transfer function form

The following simplifications based on the experimental set-up have been assumed:

- Air infiltration into the test room is negligible.
- The PAS system eliminates the heat flux through the test cell envelope.

Taking these assumptions into account, the indoor–outdoor heat exchange is possible only through the test component (see Fig. 3) and the following steady-state equation can be written for the energy balance of the test room:

$$P_{\mathrm{aux}} = UA(T_{\mathrm{i}} - T_{\mathrm{e}}) - gAG_v. \qquad (35)$$

Considering the test objectives (Section 8), the simplifications assumed and the available data (Section 8.3), a linear ARX model has been selected to estimate the parameters, assigning the following input and output variables:

- Input: P_{aux}, G_v.
- Output: T_{i}, T_{e}.

The physical constraint imposed to obtain the physical parameters from the ARX model is that the steady-state (35) and the ARX model obtained, when all its input and output are constant, must be coincident, and hence the steady-state physical parameters are obtained simply by comparing these two equations [11].

Parameters from the ARX model were estimated using MATLAB and its System Identification Toolbox.

8.4.2. Model 1: linear model in transfer function form

Assume that:

- Air infiltration into the test room may not be negligible.
- The PAS system does not completely eliminate the heat flux through the test cell envelope.
- All heat exchange between the test room and its surroundings are linear.

To take these effects into account, T_{sr}, T_{ali} and T_{ale} are included in the model (see Fig. 3), and (35) must be modified as follows:

$$P_{\mathrm{aux}} = UA(T_{\mathrm{i}} - T_{\mathrm{e}}) - gAG_v + H_{\mathrm{sr}}(T_{\mathrm{i}} - T_{\mathrm{sr}}) + H_{\mathrm{ali}}(T_{\mathrm{i}} - T_{\mathrm{ali}}) + H_{\mathrm{ale}}(T_{\mathrm{i}} - T_{\mathrm{ale}}). \qquad (36)$$

Using the same criteria as in Section 8.4.1, a linear ARX model has been selected to estimate the parameters with the following set of input and output variables:

- Input: P_{aux}, G_v.
- Output: T_{i}, T_{e}, T_{sr}, T_{ali}, T_{ale}.

The ARX model parameters were estimated using MATLAB and its System Identification Toolbox, and the required physical parameters were calculated from the ARX model, as in Section 8.4.1, but considering the steady-state equation (36) instead of (35).

8.4.3. Model 2: 'Nonlinear model' in transfer function form

Assume that:

- Air infiltration into the test room may not be negligible.
- The PAS system does not completely eliminate the heat flux through the test cell envelope.
- The effect of the infrared radiation from the indoor surface of the test room must be taken into account.

Model 1, in Section 8.4.2, is now modified by incorporating an auxiliary quantity defined as $y = (T_{\mathrm{ali}} + 273)^4$ and the model is considered linear although nonlinear in the transformed input variable.

Eq. (36) is modified as follows to take this effect into account:

$$P_{\mathrm{aux}} = UA(T_{\mathrm{i}} - T_{\mathrm{e}}) - gAG_v + H_{\mathrm{sr}}(T_{\mathrm{i}} - T_{\mathrm{sr}}) + H_{\mathrm{ale}}(T_{\mathrm{i}} - T_{\mathrm{ale}}) + \Theta(T_{\mathrm{ali}} + 273)^4. \qquad (37)$$

The steady-state energy balance in the test room is assumed to call for a term depending on the infrared radiation from the indoor surface temperature, and assuming that the only knowledge of this term is that it is dependent on $(T_{\mathrm{ali}} + 273)^4$. The term $\Theta(T_{\mathrm{ali}} + 273)^4$ completes the energy balance.

Following the same criteria as in Sections 8.4.1 and 8.4.2, a linear ARX model was selected to estimate the parameters, with the following set of input and output variables:

- Input: P_{aux}, G_v.
- Output: T_i, T_e, T_{sr}, T_{ale}, $(T_{ali}+273)^4$.

The parameters from the ARX model were estimated using MATLAB and its System Identification Toolbox, and the required physical parameters were calculated from the ARX model, as in Sections 8.4.1 and 8.4.2, but considering the steady-state equation (37) instead of (35).

Notice that nonlinear effects are not being considered for the component, but the test cell envelope.

8.4.4. Model 3: nonlinear model in continuous time state space form

Assume that:

- Air infiltration into the test room may not be negligible.
- The PAS system does not completely eliminate the heat flux through the test cell envelope.
- The effect of infrared radiation from the indoor surface of the test room must be taken into account.
- The indoor air temperature in the test room and the temperature of the corresponding sensor are slightly different.

In this case a detailed state space model, based on Model 2, and described by Eqs. (38)–(44), is considered. The main difference comparing with Model 2 is that Model 3 is a state space model so that the states of the system can be modelled in detail. The nonlinear features suggested in Model 2 are implemented in Model 3 by modelling of the following states:

- T_1, representing the indoor air temperature.
- T_5, representing the temperature of the sensor measuring the indoor air temperature. This state takes into account exchange of infrared radiation between the sensor and its surroundings.
- T_2, representing the indoor surface temperature, taking into account exchange of infrared radiation between this surface and its surroundings.
- T_{mr2} and T_{mr5}, defined as auxiliary states, representing the mean radian temperatures seen from the indoor surface and the indoor temperature sensor, respectively.

The dynamics is described by the following set of SDEs:

$$C_1\frac{dT_1}{dt} = UA(T_e - T_1) + gA(1-\alpha)G_v + P_{aux} + H_{ali}(T_2 - T_1) + H_{sr}(T_{sr} - T_1) + H_{ale}(T_{ale} - T_1) + \sigma_{11}\frac{dw}{dt}, \quad (38)$$

$$C_2\frac{dT_2}{dt} = gA\alpha G_v + H_{ali}(T_1 - T_2) + k_{mr2}(T_{mr2}^4 - T_2^4) + \sigma_{22}\frac{dw}{dt}, \quad (39)$$

$$\frac{dT_{mr2}}{dt} = k_{mr2}(T_2^4 - T_{mr2}^4) + \sigma_{33}\frac{dw}{dt}, \quad (40)$$

$$\frac{dT_{mr5}}{dt} = k_{mr5}(T_5^4 - T_{mr5}^4) + \sigma_{44}\frac{dw}{dt}, \quad (41)$$

$$\frac{dT_5}{dt} = H_{15}(T_1 - T_5) + k_5(T_{mr5}^4 - T_5^4) + \sigma_{55}\frac{dw}{dt}, \quad (42)$$

and the data observed are described by the following measurement equations:

$$T_i = T_5 + e_5, \quad (43)$$

$$T_{ali} = T_2 + e_2. \quad (44)$$

The parameters are estimated using CTSM [1].

Notice that this is an example of a model belonging to the class of nonlinear stochastic state space models as introduced in Section 2.1.

Table 1
Summary of input and output variables in the models considered

Models	Input	Output
Model 0	P_{aux}, G_v	T_i, T_e
Model 1	P_{aux}, G_v	T_i, T_e, T_{sr}, T_{ali}, T_{ale}
Model 2	P_{aux}, G_v	T_i, T_e, T_{sr}, T_{ale}, $(T_{ali}+273)^4$
Model 3	P_{aux}, G_v, T_{sr}, T_e, T_{ale}	T_i, T_{ali}

Table 2
Summary of obtained results for UA and gA

Models	UA (W/K)		gA (W/K)	
	Series 6	Series 7	Series 6	Series 7
Model 0	6 ± 1	1.8 ± 0.2	5 ± 1	0.57 ± 0.07
Model 1	9 ± 2	4.1 ± 0.6	4 ± 3	0.78 ± 0.07
Model 2	6.2 ± 0.1	6.0 ± 0.5	0.90 ± 0.02	0.91 ± 0.03
Model 3	6.84 ± 0.08	7.0 ± 0.4	0.94 ± 0.03	0.95 ± 0.05

8.5. Results

The results are summarized in Tables 1 and 2 and Figs. 8–9. The validity of these results has been analysed taking into account their consistency, and also by statistical analysis of the residuals of each considered model as shown in the following sections.

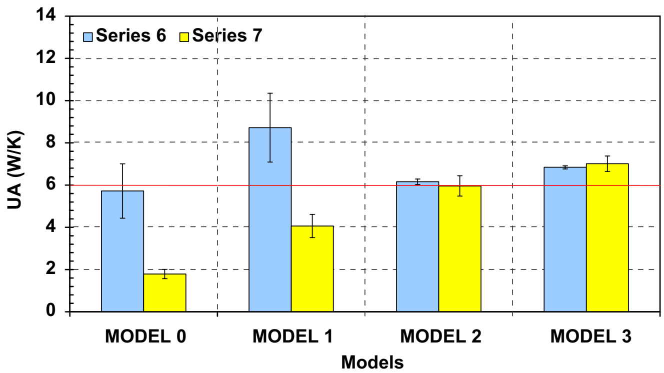

Fig. 8. Estimated *UA* for all analysed models for the two series of data.

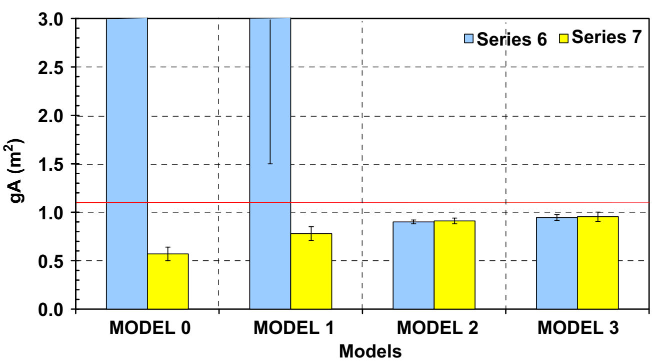

Fig. 9. Estimated *gA* for all analysed models for the two series of data.

8.5.1. Consistency of results

Consistency of results was verified by comparing the results obtained using different data sets, and also by comparing the estimated parameters with the values estimated by other procedures.

The particular reference values used for comparison are:

- $UA = 5.91\,\mathrm{W/K}$ estimated based on the properties of the materials used to construct the component according to [12].
- gA between 1.1 and $1.2\,\mathrm{m}^2$, taking into account the glazing area and g values included in [9], which reported g values of 0.66 and 0.69 at normal incidence obtained using WINDOW4 [13] and WIS [14], respectively.

Deviations from these reference values are possible due to several effects, such as thermal bridges, differences between the actual properties of the materials used and their tabulated values, accuracy in the measurements of the glazing area, etc.

The consistency of each model is discussed below:

- Models 0 and 1: Both gave very unrealistic parameter estimates as observed in the tables and graphics, identification errors are too high and results obtained using different data sets are very different.
- Model 2: There is a noticeable improvement in the estimated parameters, but the residuals are not a white noise sequence. Results seem realistic, identification errors are lower and results obtained using different data sets are similar.
- Model 3: The parameter estimates seem to be very realistic and residual whiteness is better.

Table 3
Mean and standard deviation of residuals considering the output T_i, for Series 7 and all models evaluated

Models	Average	Standard deviation
Model 0	0.001196	0.0647
Model 1	0.001672	0.0590
Model 2	0.000043	0.0580
Model 3	0.000144	0.0611

8.5.2. Analysis of residuals

In this section the validity of the considered models is analysed and compared taking their residuals into account. Series 7 was used for the residual analysis. Although the models evaluated have different output variables, T_i is the only output which is used in all models, and therefore only this output was considered for comparisons.

- Table 3 shows the mean of residuals in the same range for Models 0 and 1, although slightly higher for Model 1, and remarkably lower for Models 2 and 3. The steady-state performance of the model improved when the infrared radiation from the indoor test room surface was considered, while other quantities added to the model did not improve the results.
- Table 3 also shows that the standard deviation of residuals is similar for all models, indicating that their dynamic performance is similar.
- The shape of the autocorrelation of residuals shown in Fig. 10 is similar for all models. However, the performance of each model is better than its previous step, with Model 3 the best.
- The scaled cumulated periodogram (Fig. 11) shows that all models behave better near the steady state, whereas Model 3 performs better over the entire range of frequencies.

9. Conclusions

This paper reviews the models traditionally used for modelling the thermal characteristics of buildings and building components. The most important differences between linear and nonlinear models are outlined. It is argued that the continuous–discrete time stochastic state space model provides a strong framework for modelling physical systems. This model is often called a grey-box model due to the strong and direct link to physics. However, linear models are used for modelling thermal characteristics more often.

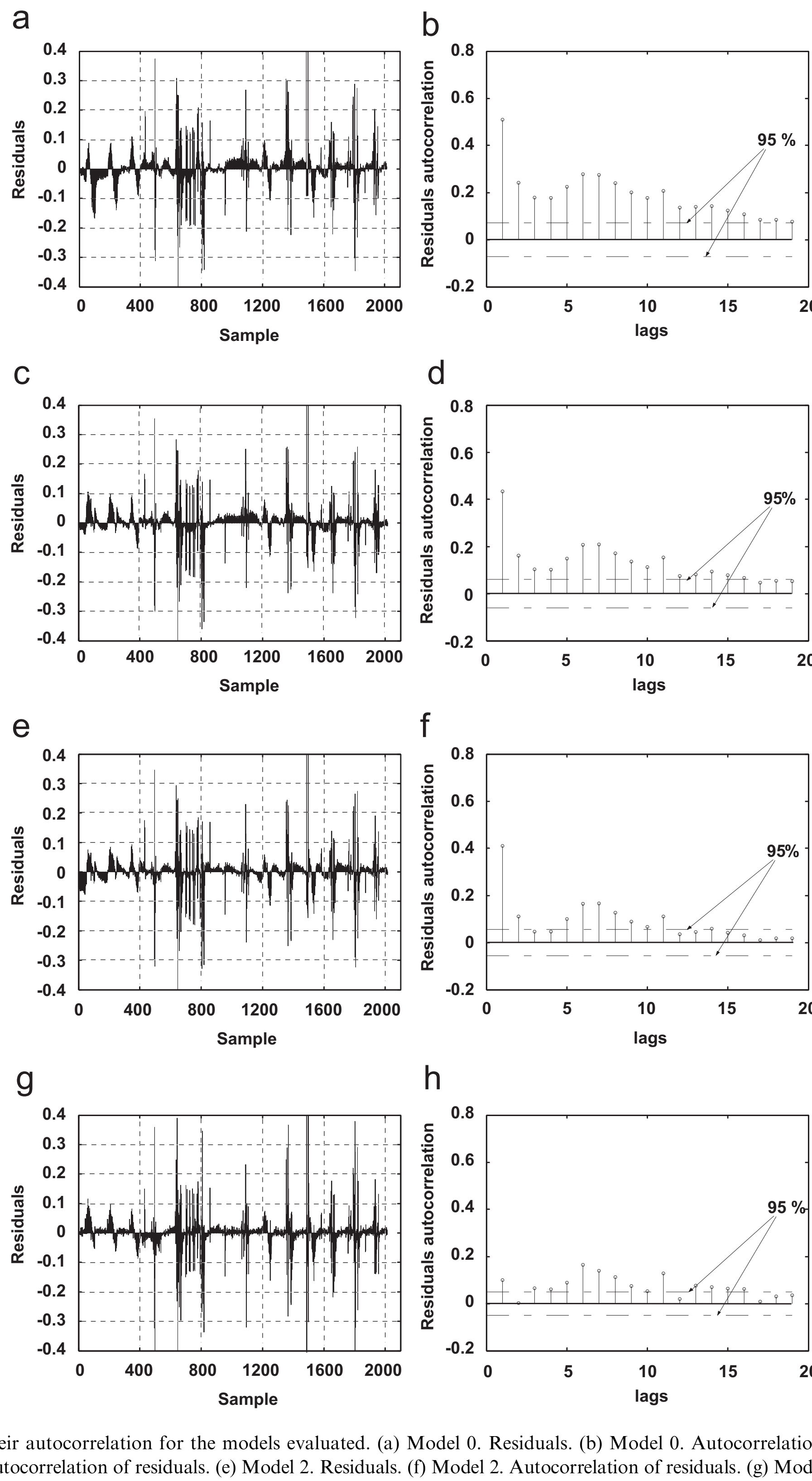

Fig. 10. Residuals and their autocorrelation for the models evaluated. (a) Model 0. Residuals. (b) Model 0. Autocorrelation of residuals. (c) Model 1. Residuals. (d) Model 1. Autocorrelation of residuals. (e) Model 2. Residuals. (f) Model 2. Autocorrelation of residuals. (g) Model 3. Residuals. (h) Model 3. Autocorrelation of residuals.

The paper gives an overview of the spectrum of linear and time-invariant models used for modelling. A special focus is put on illustrating the relationships between various linear models.

Finally, a dynamic test of a building component, carried out in a PASLINK test cell, is considered as a practical case. Both linear and nonlinear models for the heat dynamics of a wall are considered and compared in the analysis.

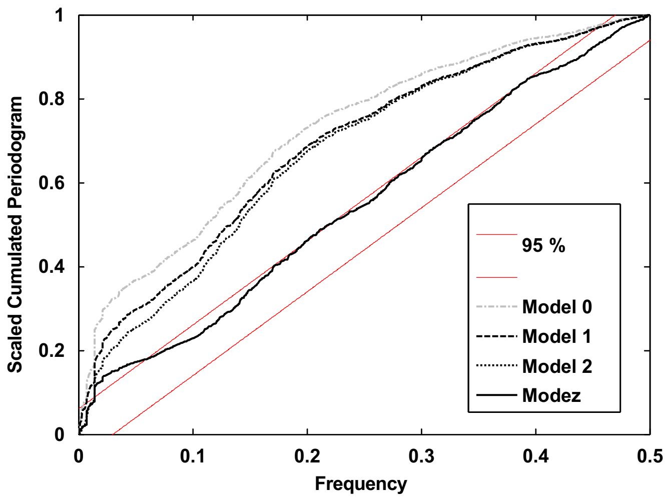

Fig. 11. Scaled cumulated periodogram.

The case study leads to the following conclusions:

- Estimations using different data sets are consistent only when infrared radiation from the indoor surface of the test room is taken into account.
- Consistency of results, regarding values estimated from properties of the materials, calls for a nonlinear model.
- The dynamic performance is almost the same for both linear and nonlinear models, however the autocorrelation function and the cumulated periodogram for the residuals show that the nonlinear state space model provides a better description than linear models.
- Although the nonlinear state space model represents the best overall performance, its residuals show a slight lack of whiteness that may be attributable to problems related to the experimental set-up, physics of sensors, the data acquisition system, etc.

It is therefore concluded that for an opaque wall component with a double-glazed window, a nonlinear model is required for an adequate description of the thermal characteristics.

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