# Improved experimental setup for observation of non-linear heat dynamics

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# Abstract

Modeling of heat dynamics of houses have been reported successful using linear dynamical models. The room they leave for improvement is – because of physical relations – believed to be partly caused by non-linear relations. As model complexity increases, detailed measurements and highly modular experiments are gaining importance in estimation of model parameters.

This paper describes test facilities and new measurement equipment in a low-energy house in arctic area. Furthermore, some of the models that will be applied are described.

# 1. Introduction

Greybox-modeling has proven to be a powerful framework for modeling of heat dynamics of buildings Madsen and Holst (1995), Andersen et al. (2000), Bacher and Madsen (2011). As building envelopes are improved, energy losses related to e.g. wind and ventilation relatively gain importance. Heat exchange is by nature a non-linear function of these phenomena. Including tese effects in the models will increase model complexity, and the increased complexity calls for better experiments to achieve good parameter estimates.

Indoor temperature measurements and recordings of consumption have been collected in a low-energy duplex house in Sisimiut, Greenland for about six years. Recently, new measurement equipment have been installed facilitating detailed observations of both heating/ventilaiton systems and indoor climate variables, and a fully integrated control system of the heating and ventilation system in the house.

This new system provides the possibilities to carry out experiments with the needed accuracy to obtain knowledge about the non-linear phenomena. The description of the heat dynamics of the building can include estimation of performance measures related to both linear phenomena such as heat conduction through the building envelope and non-linear functions such as heat loss related to wind speed, and opening of doors/windows.

Furthermore, the building is equipped with solar panels which have been used for heating of water for domestic use, and a new buffer tank has been installed. This allows for energy storage and thus strategies for optimized energy storage/dispatch can be tested.

### 2. A low-energy test-building in polar climate

In 2005, Technical University of Denmark inaugurated a low-energy house in Sisimiut, Greenland. Large temperature differences between indoors and outdoors throughout long winters and the very different distribution of sunlight over the year compared to areas where low-energy houses are usually built, make the location very interesting for modeling and testing purposes. The house is a duplex house symmetrically partitioned in two apartments of which one is inhabited by a family, the other used for presentation and experimental purposes. This partitioning facilitates good observations of influences of human behavior on heat dynamics and consumption.



Figure 1 The test-house is located in Sisimiut, Greenland, just North of the Arctic Cirle



Figure 2 The test house in Sisimiut, Greenland.

The test-house is highly insulated and equipped with a solar heating system, heat recovery, and energy efficient windows. It was designed as a low-energy house in the sense that it was planned to consume half the energy permitted by the building code. This calculation was based on the Greenlandic building code of 2006 adapted to accomodate for the advantage of heat recovery which is not common in Greenland. The house stands as an example of a combination of modern and low-energy building design on rocky ground. See Figure 2 for a picture and Figure 3 for a cross-section sketch of the house.

The gross floor area of the house is  $197 \text{ m}^2$  which is divided into two symmetric apartments, see Figure 4. The house has a common entrance hall, broiler room, and control room for heating and ventilation systems. The house is built upon a wood frame construction which is designed in a way such that thermal bridges are avoided in corners. Walls are insulated with 300 mm of mineral wool, floor, and the basement with 350 mm of mineral wool.

During the first years of operation, the house showed problems living up to the high expectations of energy performance. In 2010 an extensive mending was carried out to overcome problems due to several construction errors and malfunctioning of the heat recovery system. Measurements from 2010-2011 indicate that the energy consumption of the house has been reduced significantly and now approaches the original expectations.

Since the inauguration of the house, consumption recordings have been stored together with measurements of indoor climate variables. The consumption recordings consist of oil and electricity consumption recordings for each apartment and for common areas. For the ventilation and heating systems all inlet and outlet flows and temperatures have been measured. Moreover, measurements have been taken of temperatures and relative humidity both indoors and in some construction parts.



Figure 3 Cross-section sketch of the house.



Figure 4 Sketch of the floor plan layout of the test house.

## 2.1. Improved measurement and control equipment

In the spring 2011, new measurement equipment and a programmable logic controller (PLC) system was installed in the house. This facilitates online and centralized scheduling and surveillance of experiments. Air temperatures in all rooms, heating and ventilation inlet and outlet temperatures and flows are measured. Moreover open/closed sensors are installed on all exterior doors and windows, and  $CO_2$  concentration is measured in the apartment not rented out.

A weather station taking meteorological measurements is installed on-site. Ambient temperature and horizontal solar radiation as well as wind speed and direction are measured. See an overview of the measurements most relevant to modeling of the heat dynamics in Table 1. Meteorological data is also available from a governmental weather station nearby.

Control of heating and ventilation systems can be based on all measurements, functions hereof, or even exogenous inputs. An overview of the state of the system can be reached on-line, and a screen dump of this is seen in Figure 5. The overview intuitively shows how the different systems are connected and interact. There are two circulation systems, illustrated by different colors of the pipes. Follow the one leaving from the boiler, it goes to the domestic hot water tank (if the return valve is open), to ventilation

Measurement	Common	Rental	Experimental
	areas	apartment	apartment
Temperature	-	All rooms	All rooms Full standard indoor temperature measurement in living room
Heating Floor heating Floor heating Ventilation afterheating	1/2	0/5 All measured t All measured t	5/5 sogether
Ventilation Central ventilation Outer doors open/closed Windows open/closed Cooker hood	2/2 No windows 0/0	All measured t 3/3 2/2 0/1	together $3/3$ 2/2 1/1
Occupancy indicators PIR sensors	0	0	Living room/kitchen, corridor
$CO_2$ concentration	0	Deactivated	Yes
Consumption Oil Electricity	1/1	All measured t $1/1$	sogether $1/1$
Solar system Total heat collection Domestic water heating Buffer water heating Heating system	Co	Only one sy ntributes to com	stem mon system
Metereological variables Ambient temperature Solar radiation Wind speed Wind direction	All a	t one common w	reather station

Table 1Overview over measurements in the house of special interest for modeling of heat dynamics of the<br/>house

after heating, and/or to floor heating. Before it comes back to the furnace, it passes through a heat exchanger. Follow the other pipe system from after (left of) the solar panel. It goes to either heating the domestic water tank or to the radiator and buffer tank when a surplus of heat from the solar panel is present. The storage tank is both loaded and unloaded from the top so that a vertical temperature gradient can be maintained in the tank.

For each apartment, another on-line picture of the system is available, see Figure 6 for the experimental apartment. This picture shows temperature measurements and open/closed states of exterior doors and windows.

#### 3. Statistical methods

Statistical model building is an iterative process, and when modeling systems of high complexity it is often most fruitful to start from a simple description and then step-by-step include new terms if they significantly improve the description of the system.

A first description of the heat loss in a building is a non-linear non-dynamical model. As the aim is to characterize the performance of different construction parts, the description include terms related to different heat losses. A linear approach to estimation of both UA and gA values is outlined in (Enfor, 2010). For simplification of the measurements, the indoor temperature is assumed to be constant. The fitted expression for the heat consumption is

$$Q_t = b_0 - \mathbf{U}\mathbf{A} \cdot T_{\mathbf{a},t} - \mathbf{g}\mathbf{A} \cdot \Phi_{s,t} + b_1 W_{\mathbf{s},t} + e_t \tag{1}$$

where  $Q_t$  is the heat consumption from t-1 to t,  $T_{a,t}$  is the ambient temperature,  $R_{0,t}$  is the solar radiation, and  $W_{s,t}$  the wind speed, all in the period t-1 to t.  $b_0$  is a constant whereas  $b_1$  depend



Figure 5 An on-line overview of some of the measurements and values used for control in the system. There are two separate pipe systems. Flow measurements are taken so that all flows are known or can be calculated.



Figure 6 An overview from "inside an apartment". Set and measured temperatures are shown together with open/closed state of windows and exterior doors, movement sensors, and  $CO_2$  concentration. The column of temperature measurements to the left are standard temperature measurements taken in the living room.

on the wind direction. UA and gA are parameters which describe the heat conductivity and the solar transmittance, respectively, of the building.  $\{e_t\}$  is a white noise process.  $b_1$  could be approximated by a linear function such that three different constants each correspond to a 120° wide regime of the wind direction. By using locally weighted estimation the estimates are adaptive, and the estimates of UA and gA become functions of time, t. In (Enfor, 2010) good results are obtained estimating these coefficients for several individual households based on simple measurements and 24 hour averages.

In order to describe the heat dynamics in detail, grey-box models can be applied. Grey-box modeling

combines the advantages of using physical knowledge about the system with statistical methods to obtain precise descriptions of the dynamics behind measurements on a physical system. Grey-box modeling typically uses stochastic differential equations and since these are based on the well-known differential equations of heat dynamics, they are by nature dynamic.

A dynamical linear model is formulated in (Bacher and Madsen, 2011):

$$dT_{i} = \frac{1}{RC}(T_{a} - T_{i})dt + \frac{1}{C}A_{w}\Phi_{s}dt + \frac{1}{C}\Phi_{h}dt + \sigma_{1}d\omega_{1}$$
(2a)

$$\tau_k = T_{i,k} + e_k \tag{2b}$$

where  $T_i$  is the indoor temperature, R is the thermal resistance of the building envelope, C is the heat capacity of the building,  $A_w$  is the effective area of the windows,  $\Phi_h$  is the heat supply from the heating system.  $\omega_1$  is a Wiener process (a white noise process in continuous time), and  $\sigma_1$  is a constant. Equation (2a) is the model equation which describes the dynamics of the system. Equation (2b) is the measurement equation and expresses in this case that the discrete-time measurements of the indoor temperature are encombered with a measurement noise,  $\{e_k\}$ . Here, the short-hand notation  $t \sim t_k$  is used.



Figure 7 Sketch of a model of the heat balance in a single room house and indications of notation used.

The test facilities available for the project are expected to enable us to observe non-linear phenomena in the heat dynamics better. Hence a general heat balance in a house is considered. Let  $\Phi$  in general denote a heat flux, and the subscripts h, v, c, s, and i denote the heating system, ventilation, conduction, solar radiation, and infiltration. Then

$$dT_{\rm inv} = \frac{1}{C_{\rm inv}} \left( \Phi_{\rm h} + \Phi_{\rm v} + \Phi_{\rm a} + \Phi_{\rm s} + \Phi_{\rm i} \right) dt + \sigma_2 d\omega_2 \tag{3}$$

expresses the interior temperature development.  $\sigma_2$  is a constant and  $\omega_2$  is a Wiener process. An overview of the terms is given in Table 2. Also see Figure 7 for an illustration of the notation.

The conduction  $\Phi_a$  through walls, roof, doors and windows is expected to be of major importance. It leads to heating of the envelope which is again cooled by convection and long-wave radiation. The energy balance in a state on the outer surface of the building envelope can be described by:

$$C_o dT_o = \frac{1}{R} (T_i - T_o) dt + hA_o (T_a - T_o) dt + A_o \alpha_o K (T_{surr}^4 - T_o^4) dt + \sigma_3 d\omega_3$$
(4)

which expresses that the thermal energy accumulated in this state equals what is conducted from the inside surface plus the convection and radiation. The last term is a continuous time white noise process. In a winter situation where the inside temperature is higher than both the ambient temperature and the surface temperature of the surroundings, the first term will be positive whereas the convection and the radiation terms will be negative.

Table 2	Overview	of	the	terms	in	Equation	(3)	
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Notation	Description			
$\Phi_{ m h}$	Heat flux from the (radiating) heating system. Transfers from radiators			
	and/or floor heating by convection and radiation. Controlled and mea- sured.			
$\Phi_{\rm v}$	Advection Heat flux through ventilation system. Calculated from mea- sured volume flow and inlet/outlet temperatures.			
$\Phi_{\mathrm{a}}$	The heat flux conducting through walls, roof, doors and windows.			
$\Phi_{\rm s}$	Solar radiation through windows. Estimated using solar radiation measurements.			
$\Phi_{\mathrm{i}}$	Heat flux through infiltration. Must be estimated. Assumed to be a non-linear function of wind speed and direction.			

The first term in Equation (4) which is driven by conduction is linear. The convection term is linear for a given convection heat transfer coefficient, h. But h is expected to be a non-linear function of wind speed and wind direction. The radiation term is non-linear in both  $T_{\rm o}$  and  $T_{\rm surr}$ . With this knowledge (4) can be re-written as

$$dT_{\rm o} = k_1 (T_i - T_o) dt + f_h (W_{\rm s}, W_{\rm d}) \cdot (T_{\rm a} - T_{\rm o}) dt + k_2 (T_{\rm surr}^4 - T_o^4) dt + d\omega$$
(5)

The challenge is now to estimate the constants,  $k_1$  and  $k_2$ , and the function  $f_h$ . In (Jiménez et al., 2008)  $f_h$  is modelled for a PV-module with an allometric function. In the present case the independence of the wind direction may be insufficient. In addition a way of modeling  $T_{\text{surr}}$  must be found. This could be as a function of  $T_a$  and the exterior relative humidity.

This non-linear extension of the linear dynamic model given in Equation (2) is only one of many possible extensions. It has been justified from physical considerations but the main criteria is the ability to describe the behavior of the system. Experiments have been carried out in the test house during the spring 2011, and then different models will be evaluated on the results.

With the software CTSM<sup>1</sup> it is possible to estimate parameters in both linear and non-linear stochastic differential equations. The software can use the prediction error method and supports both maximum likelihood and maximum a posteriori estimation (Kristensen et al., 2004).

# 4. Conclusions

Statistical modeling of heat dynamics is a strong tool for characterization of and improving energy performance of buildings. Promising results have already been obtained by applying linear dynamic models on heat dynamics in buildings. Test facilities in arctic area have been described, and the advantages of these in relation to non-linear modeling have been discussed. Finally, some examples on modeling, non-dynamic and dynamic, linear and non-linear, have been given.

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<sup>&</sup>lt;sup>1</sup>http://www2.imm.dtu.dk/~ctsm/