

## **FORECAST GENERATION FOR REAL-TIME CONTROL OF URBAN DRAINAGE SYSTEMS USING GREYBOX MODELLING AND RADAR RAINFALL**

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We present stochastic flow forecasts to be used in a real-time control setup for urban drainage systems. The forecasts are generated using greybox models with rain gauge and radar rainfall observations as input. Predictions are evaluated as intervals rather than just mean values. We obtain satisfactory predictions for the smaller catchment but rather large uncertainties for the bigger catchment where the applied storage cascade seems too simple. Radar rainfall introduces more uncertainty into the flow forecast model estimation. However, the radar rainfall forecasts also result in a slightly improved point prediction of flows which we aim to exploit with a modified estimation approach in the future.

### **INTRODUCTION**

Recently, a new real-time control setup has been installed for one of the two major sewer catchments in the Copenhagen area ([5]). In this setup, control decisions are based on an optimization over predicted inflows to basins. The predictions are generated using lumped models for every subcatchment.

A clear improvement of the control decisions is expected, if forecast uncertainties can be described. The predicted inflow to a basin may e.g. just fill up the basin whereas, due to the predictive uncertainty, there is a high risk of overflow. Control decisions considering and not considering uncertainties may well be complementary in such a case. To quantify forecast uncertainties in the control framework, we intend to use greybox models. These are based on physical principles but include a stochastic term and are therefore considered useful for data driven forecasting including predictions of uncertainties.

We evaluate flow forecasts generated by greybox models for two sample catchments in the Copenhagen area. The models are estimated based on runoff volume forecasts over a horizon of 100 min. Rain gauge measurements and radar rainfall are used as input to evaluate the quality of the different data sources and the effect of rainfall forecasts generated from the radar observations.

### **DATA**

#### **Sample Catchments**

Two catchments in the Copenhagen area are considered in this study. The Ballerup catchment has a total area of approx. 1.300 ha. It is mainly laid out as separate sewer

system but has a small combined part and shows strong influences from rainfall dependent infiltration and misconnection of stormwater.

The Damhusåen catchment is located close to Ballerup but drains to a different treatment plant. We consider the northern part of the catchment with a total area of approx. 3.000 ha. The catchment is laid out as a combined sewer system and consists of several subcatchments with a longest flow path of approx. 10 km.

An overview of the catchments can be seen in Figure 1. Flow measurements in 5 min resolution are available at the outlets of both catchments. Predictions are generated for both outlets and compared to the observations at 10 min resolution where the measurements within an interval are averaged.

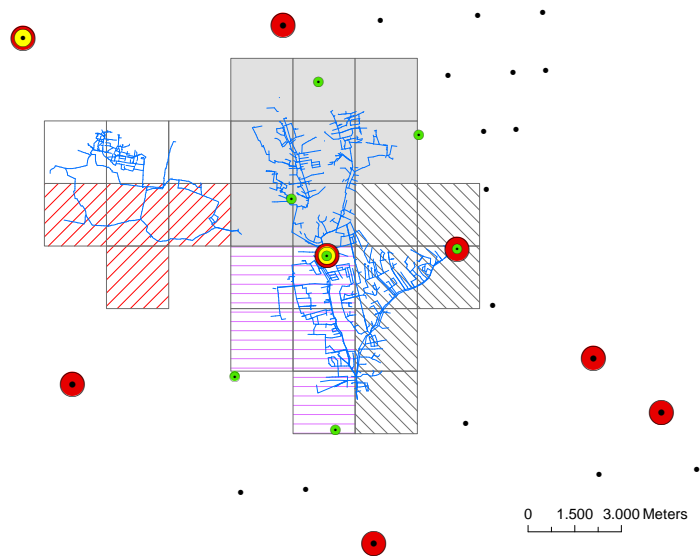


Figure 1. Ballerup (left) and northern Damhusåen (right) catchments with C-band radar pixels (2x2km) colored by subcatchments and location of rain gauges (red – used in radar calibration, yellow – used as input to Ballerup model, green – used as input to Damhusåen model, black – other gauges)

### Rainfall Measurements

Observations from tipping bucket rain gauges from the Danish SVK network ([8]) are available in the considered catchments. Rainfall measurements are available at 1 min intervals and averaged to 10 min time steps that are used for forecasting. In the rain gauge based forecast models we use 2 and 7 gauges located within or close to the catchment borders as input for the Ballerup and Damhusåen catchments, respectively (Figure 1).

The Danish weather service operates a C-band radar in Stevns approx. 45 km south of the considered catchments. Measurements from this radar are available in 10 min resolution. Figure 1 shows the location of the catchments within the utilized C-band radar pixels. An X-band radar is also located in Hvidovre close to the catchment borders.

However, data from this radar could not be utilized due to problems in the operation of the device.

### Calibration period

We use a period of 2.5 months from 25/06/2010 until 6/09/2010 for estimating the forecast models. The period contains several typical rain events that can be considered relevant for real time control (Figure 2). Further, the measurements contain no major gaps in this period.

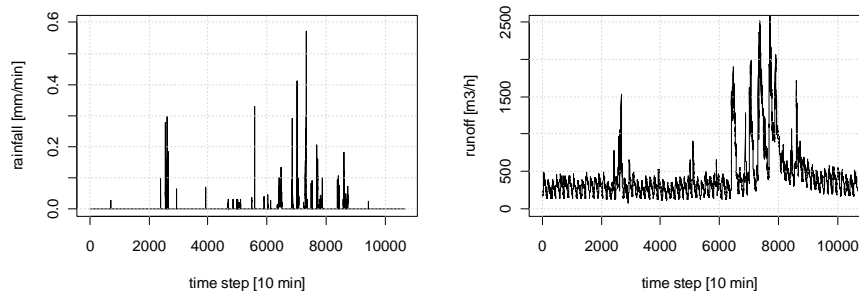


Figure 2. Areal mean of rain gauge observations and flow measurements for the Ballerup catchment in the estimation period

## METHODS

### Radar rainfall calibration

C-band radar measurements are provided as reflectivities. A direct conversion to rain intensities is commonly considered problematic. A methodology to calibrate the radar measurements to gauge observations has therefore been developed at Aalborg University and is applied here.

In a first ‘static’ step, the rain intensities derived from the radar are adjusted such that they on average over the calibration period and in the considered area give the same rainfall depths as the rain gauge measurements. In a second ‘dynamic’ step the radar rain intensities are again adjusted, this time at every time step to match the rain gauge measurements in the calibration area. We refer to [2] and [3] for a detailed description of the calibration methodology.

In the calibration we use the rain gauges shown in Figure 1. The calibration is performed with only 7 gauges distributed in the Copenhagen area as one of the main objectives for using radar rainfall measurements is to derive rain intensities using as small a number of ground measurements as possible (Figure 1).

### Stochastic Flow Forecasting

As mentioned before, we use greybox models to generate flow forecasts for the catchments. In the basic setup we use a cascade of 2 storages with one rainfall input. This setup has been extensively tested for the Ballerup catchment but may be too simple for the

Damhusåen catchment. As we are mainly interested in investigating the effects of different rainfall inputs on the forecasts, we still apply this most simple setup.

$$d \begin{bmatrix} S_{1,t} \\ S_{2,t} \end{bmatrix} = \begin{bmatrix} A \cdot P + a_0 - \frac{1}{K} S_{1,t} \\ \frac{1}{K} S_{1,t} - \frac{1}{K} S_{2,t} \end{bmatrix} dt + \begin{bmatrix} \sigma(S_{1,t}) \\ \sigma(S_{2,t}) \end{bmatrix} d\omega_t \quad (1)$$

$$\log(Q_k) = \log\left(\frac{1}{K} S_{2,k} + D_k\right) + e_k \quad (2)$$

Eq. 1 is termed system equation where  $S_1$ ,  $S_2$  correspond to the storage states,  $A$  to the sealed catchment area,  $P$  to the rain intensity,  $a_0$  to the mean dry weather flow and  $K$  to the travel time constant. The uncertainty of model predictions is captured by the Wiener process  $d\omega_t$  with incremental variance  $\sigma$ . The variance depends on the current states, so a Lambert transform is applied and the estimation performed with transformed states ([1]).

States and flow measurements are related in the observations equation (Eq. 2).  $Q$  corresponds to the observed flow values,  $D$  describes the variation of the dry weather flow using trigonometric functions and  $e$  corresponds to the observation error with standard deviation  $\sigma_e$ .

We refer to [1] and [9] for a detailed description of the modeling principles which involve a minimization of the one-step ahead flow prediction error using the open source software CTSM. However, we here estimate the model parameters by minimizing the error between a 10 step (100 min) ahead volume prediction and the corresponding observations at each time step to improve the models forecasting ability. Forecasts and uncertainties are generated by repeated updating in the Kalman Filter setup for the stochastic model making assumptions on the rainfall input as described below and using the previous predicted value as updated value for the next time step. This setup provides predictions and variances of the predicted values and we generate 95 % - prediction intervals from these.

We use different setups for including the rainfall measurements into the forecast models.

- A. Area mean – the rainfall is assumed constant over the whole catchment and inputs from gauges or radar pixels are averaged.
- B. Integrated subcatchment – the catchment is divided in subcatchments (Figure 1), a sealed area is estimated for every subcatchment, but only one storage cascade is used and all inputs are fed into the first storage.
- C. Distributed subcatchments –for the (bigger) Damhusåen catchment we aim to evaluate the use of spatial resolution in forecasting with dynamically calibrated radar data. Every subcatchment has a cascade of 2 storages of its own and the outflows from the northern and eastern subcatchments are inputs to the western subcatchment.

When generating a flow forecast one needs to make an assumption on the rainfall during the forecast horizon. We use different approaches for rain gauge and radar rainfall input:

- I. Local linear trend – a trend line is fitted to the rain gauge intensities in the past 100 min and then extrapolated over the forecast horizon
- II. CO-TREC – we use rain intensities forecasted from the radar data provided by Aalborg University ([2])

Evaluation of the forecasts is performed on the basis of predicted runoff volumes over the forecast horizon of 100 min, as these are the relevant variable for control of the system. We evaluate only forecast values generated during wet weather (predicted volume greater than the dry weather peak of approx. 1000 m<sup>3</sup>/100 min for the Ballerup catchment and 3300 m<sup>3</sup>/100 min for the Damhusåen catchment). The following measures are used in the evaluation and should be minimized:

- RMSE – root mean square error between point prediction and observed runoff volumes
- Reliability (Rel) – percentage of observations not contained in a 95 % prediction interval. Ideally, this value corresponds to 5 %, lower values suggest an overfitted model, higher values an unreliable model
- Sharpness (Sh) – average width of the 95 % prediction interval
- Skill score (Sk) 
$$Sk = Sh + \frac{2}{0.05 \cdot N} \sum_i (U_i + L_i) \quad (3)$$

where  $N$  is the number of wet weather observations and  $U_i$  and  $L_i$  are the distances of the  $i$ -th observation from the upper / lower prediction interval (over-/ undershoots).  $U_i$  and  $L_i$  are 0 if the observation is contained in the prediction band.

We refer to [6] for a derivation of performance criteria for prediction intervals.

## RESULTS

Table 1 shows examples of estimated model parameters for the two catchments using rain gauge and statically calibrated radar rainfall data as input. As the estimation aims at minimizing the mean error between predicted and observed runoff volumes, the estimated uncertainties are not very meaningful, in particular for the models with (the more uncertain) radar input.

Table 1. Estimated parameters for mean area rainfall models (type A) with rain gauge and statically calibrated radar input (estimated and fixed uncertainties). The state uncertainties  $\sigma_{z1}$  and  $\sigma_{z2}$  correspond to standard deviations of Lamberti transformed states  $S_1, S_2$

Model	K [h]	$a_0$ [m <sup>3</sup> /h]	A [ha]	$\sigma_{z2}$	$\sigma_{z1}$	$\sigma_e$
Ballerup gauge	4.1	306.7	79.0	$e^{-0.66}$	$e^{-4.14}$	$e^{-13.4}$
Ballerup radar	8.9	303.4	82.1	$e^{0.35}$	$e^{-2.62}$	$e^{-13.5}$
Ballerup rad. fixed	8.9	297.6	81.3	$e^{-0.66}$	$e^{-3.56}$	$e^{-13.5}$
Damhusåen gauge	3.3	900.1	652.0	$e^{0.63}$	$e^{-3.32}$	$e^{-13.2}$
Damhusåen radar	2.9	1059.4	202.4	$e^{0.42}$	$e^{-4.77}$	$e^{-13.5}$
Damh. rad. fixed	7.8	940.0	368.7	$e^{-0.50}$	$e^{-3.57}$	$e^{-13.5}$

Table 2. Forecast evaluation for mean area rainfall models (type A) with rain gauge and statically calibrated radar input (freely estimated and fixed uncertainties). Values are based on predicted runoff volumes in m<sup>3</sup> over a prediction horizon of 100 min

Model	RMSE	Rel	Sh	Sk
Ballerup gauge (A I)	103	12 %	682	1804
Ballerup radar (A II)	98	1 %	3201	3225
Ballerup rad. fixed (A II)	99	7 %	1134	1824
Damhusåen gauge (A I)	998	10 %	6843	14306
Damhusåen radar (A II)	1076	29 %	2438	28306
Damh. rad. fixed (A II)	999	16 %	4727	22256

An extended Kalman filter is applied in the modeling process, so the storage states can be updated to give a better description of the observed flows in the observation equation. The extent of the updating depends on the ratio of uncertainties of observation and states (see e.g. [4]). In the optimization the standard deviations are then determined in such a way, that the storage states are updated as good as possible from the last observation to give a better prediction of the mean value. Looking at the prediction intervals, we obtain too big predictive variances for the Ballerup catchment (Rel<<5%) and too small variances for the Damhusåen catchment (Rel>>5%) resulting in too big, overreliable and too narrow, unreliable prediction intervals, respectively (Table 2). This effect is particularly evident for the models with radar input.

In a first approach we fixed the uncertainties for all models with radar input by trial and error to give somewhat reliable prediction intervals with reasonable sharpness (Table 1 – models Bal rad. fixed and Damh. rad. fixed). In future works this problem will be handled by using an objective function in the estimation that accounts for the quality of the prediction intervals rather than the mean value of the prediction (e.g. skill score shown above).

Table 3 and Table 4 quantify the quality of the forecasts generated for the calibration period with different model types. For the reference models with rain gauge input (1a, 1b, 2a, 2b) we can see that the prediction intervals capture less than the intended 95 % of the observations (Rel>5%). During parameter estimation the future rainfall inputs are assumed known and introduce a too small uncertainty as compared to the forecast setting with unknown rainfall input. This problem may be solved by using unknown rainfall inputs during parameter estimation, however at the cost of more uncertain model parameters.

For model 1b we can again see the result of the objective function, minimizing the average error rather than optimizing the prediction interval. We obtain a low RMSE and as a result a narrow prediction interval which is too narrow to match the observations. Looking at the forecast results using radar data, we can first see that there seems to be no major improvement in the forecast quality when using dynamically calibrated radar data instead of statically calibrated data (comparing models 1c vs. 1d and 2c vs. 2d). This contradicts the results obtained previously for X-band radar data ([7]). A reason for this

may be that the spatially distributed hydrodynamic model applied in [7] is more sensitive to the quality of rainfall inputs than the lumped model for flow predictions.

Table 3. Forecast evaluation for the Ballerup catchment. Uncertainties for the radar inputs were fixed. Values are based on predicted runoff volumes in m<sup>3</sup> over 100 min.

Model	RMSE	Rel	Sh	Sk
1a Gauge mean area rainfall (A I)	103	12 %	682	1804
1b Gauge integrated subcatchment (B I)	97	48 %	232	3562
1c Radar stat. mean area rainfall (A II)	99	7 %	1134	1824
1d Radar dyn. mean area rainfall (A II)	96	7 %	1149	1792
1e Radar dyn. integr. subcatchment (B II)	97	7 %	1176	1793

Table 4. Forecast evaluation for the Damhusåen catchment. Uncertainties for the radar inputs were fixed. Values are based on predicted runoff volumes in m<sup>3</sup> over 100 min.

Model	RMSE	Rel	Sh	Sk
2a Gauge mean area rainfall (A I)	998	10 %	6843	14306
2b Gauge integrated subcatchment (B I)	1061	12 %	5688	15031
2c Radar stat. mean area rainfall (A II)	999	16 %	4727	22256
2d Radar dyn. mean area rainfall (A II)	988	16 %	4593	21822
2e Radar dyn. integr. subcatchment (B II)	989	16 %	4617	21925
2f Radar dyn. distrib. subcatchment (C II)	1038	17 %	4709	23534

The effect of radar rainfall forecasts on the flow forecasts is visible in the somewhat reduced RMSE values for the models with radar input and should also lead to better prediction intervals when estimating the models with an improved objective function. Spatial resolution does in none of the cases introduce significant forecast improvements over the models assuming mean area rainfall. Again, distributed models may yield different results than the lumped models applied here.

We have evaluated the ratio of sharpness and predicted volume value for the models with mean area rain gauge input. In the Ballerup catchment we obtain values between 30 and 100 % and an average of 37 % during wet weather. In the Damhusåen catchment, values vary between 50 and 250 % with an average of 82 % during wet weather, indicating that the model structure may be too simple for this catchment.

## CONCLUSIONS

It is clear from the modeling results that the predictive quality depends strongly on the suitability of the model for the catchment. For the smaller Ballerup catchment a simple storage cascade provides reasonable predictions with a level of uncertainty that is most likely useful for control purposes. In the Damhusåen catchment, the storage cascade is too much of a simplification and other effects such as overflows need to be taken into account.

There is no big improvement from using dynamically calibrated radar rainfall data over statically calibrated data in the considered cases. The same holds true for spatial resolution in the model. Radar rainfall forecasts appear to improve the mean value of the prediction to some extent. We will likely be able to exploit this effect also for the prediction intervals when optimizing the models with respect to interval instead of mean values. This is going to be the next step of work before applying the models for forecasting in several other catchments to support real time control.

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