# ARX MPC for people with type 1 diabetes

**Dimitri Boiroux, Daniel A. Finan, John Bagterp Jørgensen, Niels Kjølstad Poulsen and Henrik Madsen**

**Abstract**

Type 1 diabetes is a chronic disease characterized by a lack of production of pancreatic insulin, consequently leading to high blood glucose concentrations (hyperglycemia). Hyperglycemia has negative health effects in the long term such as eye, nerve, and kidney disease. Exogenous insulin must be injected to keep the blood glucose in the normoglycemic range (approximately 60 – 140 mg/dL, or 3.3 – 8 mmol/L). However, the dosing of exogenous insulin must be done carefully, because low blood glucose concentrations (hypoglycemia) can have immediate and severe consequences like insulin shock, coma, or even death. Currently, insulin administration is performed by the subject with type 1 diabetes based on infrequent glucose measurements (in the form of finger-sticks), often resulting in an unsatisfactory blood glucose control.

An artificial pancreas is a medical device that injects exogenous insulin automatically in order to regulate the glucose concentration. Blood glucose measurements are obtained from a continuous glucose monitor (CGM). Insulin is administrated either continuously through an insulin pump, or at discrete times using an insulin pen. A control algorithm uses previous glucose measurements and insulin injection information to compute the optimal insulin administration for the current conditions.

We use model predictive control (MPC) to compute the optimal insulin administration for 20 virtual type 1 diabetes subjects. The system (i.e., subject) has one manipulated input (insulin infusion rate), one disturbance input (carbohydrate meals), and one measured output (blood glucose concentration). The subject is represented by a system of nonlinear differential equations describing the dynamic effects of insulin and meals on blood glucose [4]. Twenty parameter sets are used in the study, each representing a different virtual subject.

The model used in the MPC is a low order autoregressive exogenous-input (ARX) model [3]. Due to both the linearity and relative parsimony of the ARX model, there is a significant amount of

subject/model mismatch in the model predictions, reflecting real-world conditions. In general, a simple ARX MPC cannot reject a step disturbance without a resulting offset; thus, the state vector is reformulated using an extended ΔARX description (E-ΔARX) [4], i.e.

$$(1-q^{-1})A(q^{-1})y(t)=(1-q^{-1})B(q^{-1})u(t)+(1-\alpha q^{-1})e(t)$$

in which $q^{-1}$ is the backward shift operator, $A$ and $B$ are polynomials, *e(t)* is a white noise process, and $0 \leqslant \alpha \leqslant 1$ is a tuning parameter.

The reference signal is time-varying, and is based on the optimal open-loop glucose profile [2]. Insulin-on-board constraints are implemented to avoid overdosing insulin. State estimation is based on a Kalman filter using the noise model described in [1] to simulate a realistic CGM.

We present the MPC results for simulations of the 20 virtual subjects with type 1 diabetes. In particular, we investigate the effects of the prediction horizon length on the control quality of blood glucose and the robustness of the solution.

**References**

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