An Investigation of CTLSM and MRQT Using Simulated Test Sequences

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1 Introduction

The purpose of this chapter is to investigate whether the estimation procedures, MRQT and CTLSM, are able to estimate the parameters of systems, which, by simulation, are provided as linear systems. For real test sequences this information is impossible to obtain, since the models considered only are approximations of the real system. The real system is a distributed system, not a lumped parameter system, and the real system is known to contain both non-linearities and to be time-variant. Hence, in order to separate the complexity of the real system from the estimation procedure, a simulation procedure is the most reasonable. By this approach we shall try to verify that the proposed estimation procedures are able to estimate parameters in linear and time-invariant systems, which are specified in continuous time.

Furthermore the investigation focus on the estimated uncertainty of the parameters as provided by the methods.

2 The Model

For the simulation study a simple lumped parameter model for a building is considered. The model and the parameters are taken from results obtained in a previous Danish estimation study for a test building.

The dominating heat capacity of the test building is located in the outer wall. For such buildings, the model with two time constants shown in Figure 1 is frequently found adequate. The states of the model are given by the temperature, T_i, of the indoor air and possibly inner part of the walls with heat capacity C_i, and by the temperature, T_m, of the heat accumulating medium, with the heat capacity C_m. H_i is the transmittance of heat transfer between the room air and the walls, while H_m is the heat transmittance between the inner part of the walls and the external surface of the walls. The input to the system is the heat supply, Q_h, and the outdoor surface temperature, T_e. By considering the outdoor surface temperature instead of the outdoor air temperature, the effect of solar radiation is taken into account.

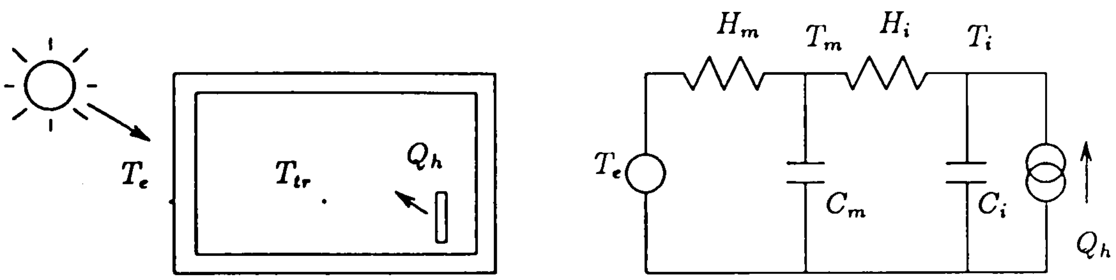

Figure 1: *A model with two time constants of the test building and the equivalent electrical network.*

In state space form the model is written,

$$\begin{bmatrix} dT_i \\ dT_m \end{bmatrix} = \begin{bmatrix} -H_i/C_i & H_i/C_i \\ H_i/C_m & -(H_i+H_m)/C_m \end{bmatrix} \begin{bmatrix} T_i \\ T_m \end{bmatrix} dt + \begin{bmatrix} 0 & 1/C_i \\ H_m/C_m & 0 \end{bmatrix} \begin{bmatrix} T_e \\ Q_h \end{bmatrix} dt + \begin{bmatrix} dw_i(t) \\ dw_m(t) \end{bmatrix}. \quad (1)$$

An additive noise term is introduced to describe deviations between the model and the true system. Hence, the model of the heat dynamics is given by the (matrix) stochastic differential equation

$$\boldsymbol{dT} = \boldsymbol{AT}dt + \boldsymbol{BU}dt + \boldsymbol{dw}(t) \quad (2)$$

where $w(t)$ is assumed to be a *wiener process* with incremental covariance matrix

$$\Sigma = \begin{bmatrix} \sigma_{1,i}^2 & 0 \\ 0 & \sigma_{1,m}^2 \end{bmatrix}. \quad (3)$$

The measured air temperature is naturally encumbered with some measurement errors, and hence the measurement equation is written

$$T_{tr}(t) = (1\ 0) \begin{bmatrix} T_i \\ T_m \end{bmatrix} + e(t) \quad (4)$$

where $e(t)$ is the measurement error, assumed to be normally distributed with zero mean and variance σ_2^2.

The following parameter values have been estimated in an earlier experiment on a test building: $H_i = 55.29$ W/K , $H_m = 13.86$ W/K , $C_i = 325.0$ Wh/K, $C_m = 387.8$ Wh/K, $\sigma_{1,i}^2 = 0.00167$ K^2, $\sigma_{1,m}^2 = 0.00978$ K^2, and $\sigma_2^2 = 0.00019$ K^2. Corresponding to these parameters, the time constants of the system are $\tau_1 = 3.03$ hours and $\tau_2 = 54.28$ hours.

3 Simulation

This paper describes some of the results from several simulations and estimations of system specified in the previous section. The main purpose is to validate the estimation procedure. By considering several simulated sequences this investigation considers both the mean values and the variances of the estimated parameters.

The input and output signals in the model are:

T_e is measured surface temperature, from a Danish test building.

Q_h the heat supply, is a PRBS (pseudo-random binary sequence) of order $n = 6$ and $T_{prbs} = 8$ hours, switching between 0 W and 300 W.

T_{tr} room temperature, simulated with the specified model.

In this study the sampling time is $T_{sampl} = 20$ minutes, and the length of one experiment is 21 days, which equals 1512 observations per simulated series. We have simulated 10, in principal, equal series, but with different realizations of the noise sequences.

4 Results

A summary of the estimation results from the 10 series is presented in Table 1 to Table 3.

Table 1 and Table 2 show results from estimations with CTLSM 2.4. In CTLSM 2.4 there is a possibility of selecting an estimation criterium which involves k-step prediction errors instead of the usual 1-step predictions errors, which theoretically enters the maximum likelihood estimation. Table 1 shows the result for 1-step case (Maximum Likelihood) and Table 2 shows the result for a 4-step estimation.

From the maximum likelihood estimates obtained with CTLSM 2.4, and shown in Table 1, it is seen that not only are the estimated parameters (and functions of parameters - UA- and CI-values) close to the simulated or true values, but also the uncertainty of the estimates (given as the standard deviation) are close to the empirical standard deviation. Hence its is concluded, that this method provides reliable uncertainty regions for the parameters, and it is possible to make test statistics for model reduction, etc.

In Table 2 the results are shown of the 4-step estimation method used on the same data. This method puts more weight on the low frequency dynamics. The parameter estimates seems to be correct, but it is clearly seen that it is no longer possible to obtain uncorrelated residuals, and the standard deviation on parameters will then be too optimistic.

Table 1: *Results from* **CTLSM 2.4** *with k-step=1. The mean autocorrelation of residuals is,* $\bar{\rho}(1) = -0.001$. x_{simul} *is the simulated/true parameters,* $\bar{x}$ *is the mean of the estimated parameters,* s_x *is the empirical standard deviation of the estimated parameters and* $\bar{s}$ *is the mean of the estimated standard deviation.*

Parameter		x_{simul}	$\bar{x}$	s_x	$\bar{s}$
H_i	[W/K]	55.29	55.83	1.786	1.579
H_m	[W/K]	13.86	13.79	0.1686	0.1860
C_i	[Wh/K]	325.0	323.6	3.663	2.925
C_m	[Wh/K]	387.8	379.3	9.860	18.08
$\sigma_{1,i}$	[K]	0.04087	0.04122	0.01299	0.01378
$\sigma_{1,m}$	[K]	0.09889	0.09597	0.03992	0.05608
σ_2	[K]	0.01378	0.01355	0.006235	0.005447
UA	[W/K]	11.08	11.06	0.09910	0.09746
CI	[MJ/K]	2.286	2.260	0.03556	0.05084

Table 2: *Results from* **CTLSM 2.4** *with k-step=4. The mean autocorrelation of residuals is,* $\bar{\rho}(1) = 0.716$. x_{simul} *is the simulated parameters,* $\bar{x}$ *is the mean of the estimated parameters,* s_x *is the empirical standard deviation of the estimated parameters and* $\bar{s}$ *is the mean of the estimated standard deviation.*

Parameter		x_{simul}	$\bar{x}$	s_x	$\bar{s}$
H_i	[W/K]	55.29	55.95	1.776	0.8034
H_m	[W/K]	13.86	13.78	0.1822	0.08815
C_i	[Wh/K]	325.0	323.1	4.198	1.682
C_m	[Wh/K]	387.8	379.0	9.899	7.698
$\sigma_{1,i}$	[K]	0.04087	0.04135	0.01214	0.01268
$\sigma_{1,m}$	[K]	0.09889	0.09537	0.03828	0.04103
σ_2	[K]	0.01378	0.01337	0.006734	0.008021
UA	[W/K]	11.08	11.06	0.1050	0.04699
CI	[MJ/K]	2.286	2.258	0.03664	0.02282

The results from estimation on the same data set using MRQT 5.2 are summarized in Table 3. The estimates are obtained from minimizing the sum of squared simulation errors.

An extra column is added to the table, containing the standard deviations corrected for the autocorrelation of the residuals, where the formula given by TNO is used.

Table 3: *Results from* MRQT 5.2 *with standard deviations, corrected for autocorrelation of residuals. The mean autocorrelation of residuals is,* $\bar{\rho}(1) = 0.978$. x_{simul} *is the simulated parameters,* $\bar{x}$ *is the mean of the estimated parameters,* s_x *is the empirical standard deviation of the estimated parameters,* $\bar{s}$ *is the mean of the estimated standard deviation and* $\bar{s}_{cor}$ *is the mean of the estimated standard deviation corrected for autocorrelation of residuals.*

Parameter	x_{simul}	$\bar{x}$	s_x	$\bar{s}$	$\bar{s}_{cor}$
H_i [W/K]	55.29	45.23	7.130	2.224	17.10
H_m [W/K]	13.86	14.81	0.8744	0.2395	1.797
C_i [Wh/K]	325.0	365.4	21.65	5.130	39.26
C_m [Wh/K]	387.8	343.0	25.09	6.809	51.64
UA [W/K]	11.08	11.07	0.1068	0.01417	0.1160
CI [MJ/K]	2.286	2.239	0.06930	0.02863	0.2163

In Table 1 and Table 2 it is seen that it has been possible to estimate the right parameters as well as the dominating characteristics (UA and CI). In Table 3 only the dominating characteristics seems to be correct.

In Table 1 the estimated standard deviations are all right. In Table 2 the estimated standard deviations are too small, and the residuals are auto-correlated. In Table 3 the estimated standard deviations are too small, and the residuals are auto-correlated. When standard deviations are corrected for the autocorrelation, only the standard deviation of the UA-value seems to be all right.

5 Conclusion

The purpose of the present paper have been to use a simulation study to investigate the estimation methods. Clearly, if the system is a linear system (as in the simulation study), both methods should provide correct parameter estimates and correct estimates of the uncertainties.

The main conclusions are:

- Both CTLSM and MRQT give correct estimates of the dominating characteristics (UA and CI) in all cases.
- Only CTLSM (k-step = 1) seems to give correct estimates of the uncertainty. For CTLSM (k-step = 4) and MRQT the obtained variances of the estimates are too small. However, using the correction for autocorrelation provided by TNO, the

estimate of the uncertainty of UA seems to be correct; but the uncertainty estimate related to CI seems to be wrong.

- For the individual dynamical parameters (H_i, H_m, C_i and C_m) only CTLSM (in both cases) seems to give correct parameter estimates.
- For the individual dynamical parameters the estimate of the uncertainty provided by CTLSM (k-step = 1) seems to be reasonable. The use of k-step = 4 seems to imply that the estimated uncertainty is too small. MRQT (without correction for autocorrelation) seems to give too optimistic uncertainty of the estimated parameters, and observed variation (s_x) is very high compared to what is seen for CTLSM. With the correction for autocorrelation provided by TNO it seems that the variances are too high.

In summary, only CTLSM with k-step = 1 gives reasonable parameter estimates as well as reasonable estimates of the uncertainty. CTLSM with k-step = 4 also gives reasonable parameter values; but too optimistic estimates of the uncertainty. MRQT seems not to give reasonable estimates of the uncertainty in general, and only the UA and the CI values seem to be correct.

From a theoretical point of view it is expected that the estimate of the uncertainty is too optimistic in the case k-step = 4 for CTLSM and for MRQT in general, since the residuals are auto-correlated (and not close to white noise). This is due to the fact that the effective number of freedoms is lesser than encounted by the estimation procedure in these cases.