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Determining reserve requirements in DK1 area of Nord Pool using a probabilistic approach

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ABSTRACT

Allocation of electricity reserves is the main tool for transmission system operators to guarantee a reliable and safe real-time operation of the power system. Traditionally, a deterministic criterion is used to establish the level of reserve. Alternative criteria are given in this paper by using a probabilistic framework where the reserve requirements are computed based on scenarios of wind power forecast error, load forecast errors and power plant outages. Our approach is first motivated by the increasing wind power penetration in power systems worldwide as well as the current market design of the DK1 area of Nord Pool, where reserves are scheduled prior to the closure of the day-ahead market. The risk of the solution under the resulting reserve schedule is controlled by two measures: the LOLP (Loss-of-Load Probability) and the CVaR (Conditional Value at Risk). Results show that during the case study period, the LOLP methodology produces more costly and less reliable reserve schedules, whereas the solution from the CVaR-method increases the safety of the overall system while decreasing the associated reserve costs, with respect to the method currently used by the Danish TSO (Transmission System Operator).
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1. Introduction

Electricity is a commodity that must be supplied continuously at all times at certain frequency. When this requirement is not fulfilled and there is shortage of electricity, industrial consumers can face the very costly consequences of outages: their production being stopped or their systems collapsed. Households will experience high discomfort and losses too. From a different point of view, service interruptions also affect electricity producers as they are not able to sell the output of their power plants. Therefore, it is of high importance that the demand is always covered. The main tool for transmission system operators to avoid electricity interruptions is the allocation of operating reserves. In practice, scheduling reserves means that the system is operating at less than full capacity and the extra capacity will only be used in case of disturbances.

The term *operating reserves* is defined in this paper as "the real power capability that can be given or taken in the operating time frame to assist in generation and load balance and frequency control" [1]. The types of reserves are differentiated by three factors: first, the time frame when they have to be activated ranging from few seconds to minutes; secondly, their activation mode, either automatically or manually; finally, by the direction of the response, upwards or downwards. Members of the ENTSO-E (European Network for System Operators for Electricity) and more specifically, the Danish TSO (Transmission System Operator), follow this classification criterion. Primary control is activated automatically within 15 s and its purpose is to restore the balance after a deviation of ±0.2 mHz from the nominal frequency of 50 Hz. Secondary control releases primary reserve and has to be automatically supplied within 15 min or 5 min if the unit is in operation. Manual reserve releases primary and secondary reserves and has to be supplied within 15 min. In Denmark, this type of reserve is often provided by CHP (Combined Heat-and-Power) plants and fast start units. The activated manual reserves are often referred as regulating power. This paper deals with the total upward reserve requirements, namely the sum of primary, secondary and manual reserves, neglecting the short-circuit power, reactive and voltage-control reserves. The result of the proposed optimization models, namely the schedule of reserves, refers to the total MW of upward reserve required. It is assumed that the reserve acts instantaneously to any generation deficit and no activation times are considered.

Currently the provision of reserve capacity in the DK1 area of Nordpool obeys the following rules, which can be found in the

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official documents issued by the Danish TSO [2]. The requirements for primary and secondary reserve are ±27 MW and ±90 MW, respectively. The provision of tertiary or manual reserves follows the recommendations in both the ENTSO-E Operation Handbook and the Nordic System Operation Agreement [3], where it is stipulated that each TSO must procure the amount of tertiary reserves needed to cover the outage of a dimensioning unit in the system (the so-called $N-1$ *criterion*), be it a domestic transmission line, an international interconnection or a generating unit. The inspection of the historical data reveals that this criterion roughly results in an amount of tertiary reserve in between 300 and 600 MW.

The methodologies discussed in this article are mainly targeted to the current structure of the Danish electricity market, where reserve markets are settled independently of and before the day-ahead energy market, implying that, at the moment of scheduling reserves, no information about which units will be online is known. At the market closure, the TSO collects bids from producers willing to provide reserve capacity, and selects them by a cost merit-order procedure. Most of the existing literature focuses on co-optimizing the unit commitment and reserve requirements at the same time; such methods however cannot be applied under the current design of the Danish electricity market.

This paper is also motivated by the increasing penetration of wind power production in Europe and, in particular, in Denmark. As a matter of fact, the commission of the European Countries has set an ambitious target such that the EU will reach 20% share of energy from renewable sources by 2020, and in Denmark the target is 30% [4]. As the share of electricity produced by renewables increases, several challenges must be faced. Non-dispatchable electricity generation cannot ensure a certain production at all times, but instead depends on meteorological factors. The stochastic nature of such factors inevitably leads to forecast errors that will likely result in producers deviating from their contracted power, thus causing the system to be imbalanced. Solutions call for methods capable of managing the uncertainty that wind power production and other stochastic variables induce into the system.

The main contributions of this paper are the following:

1. A probabilistic framework to determine the total reserve requirements independently to the generation power schedule in a power system with high penetration of wind production. The reserve levels in Denmark are currently computed by deterministic rules such as allocating an amount of reserve equal to the capacity of the largest unit online [2,3]. Another example is the rule used in Spain and Portugal, where the upward reserve is set equal to 2% of the forecast load plus the largest unit in the system. These rules are designed for systems with very low penetration of renewable energy and fairly predictable load, where the biggest largest need for reserve capacity arises from outages of large generation units. With the increasing share of renewables (and decentralized production in general) in the generation portfolio, renewables will naturally have a larger influence on the system imbalance. Hence, the non-dispatchable and uncertain nature of these plants needs to be accounted for when reserve power is scheduled [5]. Previous studies perform a co-optimization of the energy and reserve markets, either in a deterministic manner [6] or in a probabilistic way [7–13]. However, these methods cannot be applied directly to the DK1 area of Nord Pool since the reserve market and the day-ahead energy market are cleared independently at different times and by different entities. The methodology in Ref. [14] is not suitable either as the COPT (Capacity Probability Table) refers to the units that are online; this information is not available to the Danish TSO at the time of clearing the reserve market.
2. A flexible scenario-based approach for modeling system uncertainty, which takes into account the limited predictability of wind and load, and plausible equipment failures. Moreover, the distributions from which the scenarios are generated are time-dependent, being the distributions of the scenarios of forecasts errors of load and wind power production non-parametric and correlated. The authors of [9] characterize the uncertainty in the system only by scenarios of wind power forecast errors. Load and wind generation uncertainty is described in Refs. [8,15] by independent Gaussian distributions and not in a scenario framework. Other authors [7,15] use outage probabilities as a constant parameter for each unit and for each hour. The authors of [14] represent the forecast error distributions of the load and wind generation by a set of quantiles, assuming both distributions are independent.
3. Equipment failures are modeled as the amount of MW that fail in the whole system due to the forced outages of generating unit. This way we can model and simulate simultaneous outages. Furthermore, the distribution of failures is dependent on time. Existing literature takes into account just one or two simultaneous failures [7,8,15] or several [14].
4. Two different methods for controlling the risk of the resulting capacity reserve schedule. The first one imposes a target on the probability of load shedding as in Refs. [7], while the second one is based on the Conditional Value at Risk of the reserve cost distribution. The latter method minimizes the societal costs, while penalizing high cost scenarios given a certain level of risk aversion.

The remaining of the paper is organized as follows. Section 2 presents two different optimization models for reserve determination. Section 3 describes the methodology to generate scenarios of load forecast error, wind power forecast error and equipment failures, which altogether constitute the input information to the proposed reserve determination models. Section 4 elaborates on the estimation of the cost of allocating and deploying reserves. Section 5 discusses the results and comments on the implications of applying the two reserve determination models to the Danish electricity market. Conclusions are summarized in Section 6.

2. Modeling framework

This section presents two formulations for determining the reserve requirements in DK1, both of them solved using a scenario-based approach. The first limits the LOLP (Loss-Of-Load Probability), while the second one minimizes the CVaR (Conditional Value at Risk) of the cost distribution of reserve allocation, reserve deployment and load shedding. Both models are meant to be run to clear the reserve market and can be used by the TSO to decide on how many MW of reserve should be scheduled. In Denmark, where the study case in this paper is focused, the reserve market is cleared previous to and independently of the day-ahead energy market. This implies that the unit commitment problem is not addressed at the time when the reserve market closes and thus neither is it in this paper.

2.1. LOLP-formulation

The objective is to minimize the total cost of allocating reserves,

$$\underset{R_i}{\text{Minimize}} \sum_{\mathrm{i}}^{\mathrm{M}} \lambda_{\mathrm{i}} R_{\mathrm{i}}, \tag{1a}$$

where R_i is a variable representing the total amount of reserve assigned to producer i and λ_i is the price bid submitted to the reserve market by this producer. M is the total number of bids. The objective (1a) is subject to the following constraints

$$R_i \le R_i^{max} \quad \forall i \tag{1b}$$

$$R^T = \sum_{i}^{M} R_i \tag{1c}$$

$$\text{LOLP} = \int_{R^T}^{\infty} f(z)\mathrm{d}z \tag{1d}$$

$$\text{LOLP} \le \beta \tag{1e}$$

$$R^T \ge 0 \quad \forall i. \tag{1f}$$

The set of inequalities (1b) indicates that the amount of reserve provided by producer i cannot be greater than its bid quantity. The total reserve to be scheduled is defined in (1c) as the sum of the reserve contribution from each producer. The probability density function of balancing requirements is represented by $f(z)$, and hence the integral from $z = R^T$ to $z = \infty$ is the probability of not scheduling enough reserves to cover the demand, namely the loss-of-load probability, defined in (1d). It is constrained by a parameter target $\beta \in [0,1]$ in Equation (1e), which is to be specified by the transmission system operator. The smaller β is, the more reserves are scheduled, as the LOLP is desired to be small. On the other hand, if β is equal to 1, no reserves are allocated at all.

The optimal solution to problem (1a) can be found analytically, under the assumption that the objective function (1a) is monotonically increasing with respect to the total scheduled reserve R^T (i.e., reserve capacity prices are non-negative) and because the LOLP is a decreasing function with respect to R^T (note that $f(z)$ is a density function and therefore, always non-negative). Indeed, under the above assumption, greater R^T implies greater costs, thus R^T is pushed as low as possible until the relation LOLP $= \beta$ is satisfied. Therefore, at the optimum, it holds that $\beta = \int_{R^T}^{\infty} f(z)\mathrm{d}z$ or similarly $1 - \beta = F(R^T)$ being $F(Z) = P(Z \le z)$ the cumulative distribution function of Z (the required reserve). Finally, since β is a given parameter, then the solution is $R^{T^*} = F^{-1}(1-\beta)$.

In practice, $f(z)$ can be difficult to estimate in a closed form; one way of dealing with this issue is to describe the uncertainty by scenarios. Let z_w be the reserve required to cover balancing needs in scenario w and π_w the associated probability of occurrence. Then the optimal solution to problem (1a) boils down to the quantile $1-\beta$ of the scenarios. In other words, let $\widehat{F}(Z) = P(Z \le z)$ be the empirical cumulative distribution function of the set of scenarios $\{z_w\}$ with $\widehat{F} : (-\infty,\infty) \rightarrow (0,1)$, then the analytical solution is $R^{T^*} = \inf\{z \in (-\infty,\infty) : (1-\beta) \le \widehat{F}(z)\}$.

Finally, we define the EPNS (Expected Power Not Served) as the expected amount of MW of balancing power needed during 1 h which cannot be covered by the scheduled reserves. It can be computed, once the total scheduled reserve R^{T^*} has been obtained, as.

$$\text{EPNS} = \int_{R^{T^*}}^{\infty} z f(z)\mathrm{d}z. \tag{2}$$

In the case where the uncertainty of reserve requirements is characterized by scenarios, the EPNS can be determined as $\text{EPNS} = \sum_{w \in S} (z_w - R^{T^*})\pi_w$, $S = \{w \in W : z_w > R^{T^*}\}$.

2.2. CVaR (Conditional value at risk) formulation

The following reserve determination model corresponds to a two-stage stochastic linear program where each scenario is characterized by a realization of the stochastic variable Z "*reserve requirements*". Variable R^T represents the amount of MW that the TSO should buy at the reserve market. In the jargon of stochastic programming, this variable is referred to as a *first stage variable*, or equivalently, as a *here-and-now* decision, i.e., a decision that must be made before any plausible scenario z_w of energy shortage is realized. This models the fact that reserve capacity is to be scheduled before the scenarios of reserve requirement are realized. For their part, the *second stage variables*, or recourse variables, r_w^T and L_w, are relative to each scenario w, and represent the deployed regulating power and the MW of shed load, respectively. Consequently, during the real-time operation of the power system, once a certain scenario w of wind power production, load and equipment failures realizes, reserve is activated r_w^T or some load is shed (L_w). In such a way, the first stage of our stochastic programming model represents the reserve availability market and the second stage represents the reserve activation market. Finally, the probability of occurrence of each scenario is denoted by π_w.

The objective function to be minimized is the CVaR$_\alpha$ of the distribution of total cost. By definition, the Value-at-Risk at the confidence level α (VaR$_\alpha$) of a probability distribution is its α-quantile, whereas the CVaR$_\alpha$ is the conditional expectation of the area above the VaR$_\alpha$. The CVaR is known to have better properties than the VaR [16] and hence, it is used in this paper. Parameter $\alpha \in [0,1)$ represents the risk-aversion of the TSO, i.e. the greater α is, the more conservative the solution will be in terms of costs. The objective is to minimize the CVaR$_\alpha$ of the distribution of the total cost:

$$\underset{R^T,R_g,r_w^T,r_{gw},L_w,\xi,\eta_w,\text{Cost}_w}{\text{Minimize}} \quad \text{CVaR}_\alpha = \xi + \frac{1}{1-\alpha}\sum_{w=1}^{W} \pi_w \eta_w \tag{3a}$$

where ξ is, at the optimum, the $\alpha-$Value at Risk (VaR$_\alpha$) and η_w is an auxiliary variable indicating the positive difference between the VaR and the cost associated with scenario w. The cost of each scenario, named Cost$_w$, is computed in (3b) as the sum of the cost of allocating and deploying reserve capacity plus the cost incurred by involuntary load shedding. The objective (3a) is subject to the following constraints:

$$\text{Cost}_w = \sum_{j=1}^{J} \lambda_j^{\text{cap}} R_j + \sum_{g=1}^{G} \lambda_g^{\text{bal}} r_{gw} + V^{\text{LOL}} L_w \quad \forall w \tag{3b}$$

$$R^T = \sum_{j=1}^{J} R_j \tag{3c}$$

$$r_w^T = \sum_{g=1}^{G} r_{gw} \quad \forall w \tag{3d}$$

$$0 \le R_j \le I_j^R \quad \forall j \tag{3e}$$

$$0 \le r_{gw} \le I_g^r \quad \forall g, w \tag{3f}$$

$$\text{Cost}_w - \xi \le \eta_w \quad \forall w \tag{3g}$$

$$r_w^T \leq R^T \quad \forall w \tag{3h}$$

$$z_w - r_w^T \leq L_w \quad \forall w \tag{3i}$$

$$0 \leq L_w, \eta_w \quad \forall w. \tag{3j}$$

The first term in Equation (3b) represents the cost of allocating R^T MW of reserve capacity. The TSO has information about the marginal cost of allocating reserves at the closure time of the reserve market, as it is given by the bids submitted by producers to the reserve market. These bids, however, are treated confidentially and hence, were not available for the study case. Consequently, we estimate a cost function for the supply of reserve capacity from the historical series of clearing prices in the Danish reserve market. Naturally, this function must be monotonically increasing. The estimation of the parameters of this function is discussed in Section 3. In order to keep formulation (3) linear, the marginal cost of reserve capacity is further approximated by a stepwise function consisting of J intervals of length I_j^R each, as indicated in (3e), and associated values λ_j^{cap}, which result from evaluating the estimated reserve cost function at the midpoint of each interval. The term $\sum_{j=1}^{J} \lambda_j^{\text{cap}} R_j$ represents thus the total cost of allocating R^T MW of reserves. Furthermore, the total allocated reserves are given by (3c). Note that the formulation would remain equal if the real bids were used instead of the estimated cost function. One could interpret λ_j^{cap} and I_j^R as the bid that producer j submit to the reserve market and R_j as the MW of reserve capacity provided this producer.

The second term of (3b) represents the reserve deployment cost. This cost is unknown at the time of clearing the reserve market and therefore, has to be estimated by as well. The estimation procedure is discussed in Section 3. Similarly as before, λ_g^{bal} can be seen as the cost of deploying r_{gw} MW of reserve in interval g and scenario w. The length of the intervals is I_g^r, as stated in (3f), having a total of G intervals. The total deployed reserve in scenario w is then given by (3d).

The third term of (3b) represents the cost of involuntary load curtailment. The parameter “Value of Lost Load” V^{LOL} expresses the societal cost of shedding 1 MWh of load. Often, the V^{LOL} is interpreted as the maximum price of upward regulation that is permitted to bid in the market, which in Denmark is 37,500 DKK/MWh or roughly 5,000 €/MWh. In Great Britain, the V^{LOL} is estimated to be from 1,400 £/MWh to 39,000 £/MWh depending on the type of consumer and the time of the year [17]. A study performed on the Irish power system indicates that, on average, the V^{LOL} is 12.9 €/KWh [18]. In this paper, a sensitivity analysis is performed to study how the parameter V^{LOL} affects the solution.

Constraint (3g) is used to linearly define the CVaR_α as in Ref. [19]. Variable η_w is equal to zero if $\text{Cost}_w < \xi$, and equal to $\text{Cost}_w - \xi$ if $\text{Cost}_w \geq \xi$; in other words, η_w accounts for the difference between the cost in each scenario and the VaR_α when such a difference is positive. Equation (3h) indicates that the deployed reserve cannot be greater than the scheduled reserves. Equation (3i) is used to define the shed load L_w (or similarly, the lack of reserve). At the optimum, L_w is equal to zero if $z_w \leq R^T$, implying that $z_w = r_w^T$; when $z_w > R^T$, then L_w is equal to the difference between the reserve requirements and the deployed reserves, namely, $z_w - r_w^T$. In this case, the deployed reserve is equal to the scheduled capacity reserve $r_w^T = R^T$.

Once the CVaR problem has been solved, one can calculate the EPNS by multiplying the lacking reserve from each scenario L_w^* at the optimum by its probability of occurrence π_w, namely $\text{EPNS} = \sum_{w=1}^{W} \pi_w L_w^*$.

3. Cost functions

This section elaborates on the estimation of the cost of allocating reserves and the cost of providing regulating power.

In practice, the bids that producers submit to the reserve market, that are used to define (3b), are available to the Danish TSO at the closure of the reserve market. Nevertheless, this information is not available to us for the case study presented in Section 5. Consequently, in order to adjust the optimization models to the available data and test the efficiency of such, the bids of producers are substituted by a cost function, being $g^R(z)$ the cost in €/MW of allocating of z MW of upward reserve. This function is built from the series of clearing prices in the Danish reserve market, which is publicly available in Ref. [20]. The price per MW of reserve capacity is assumed to be quadratic for simplicity, in particular, of the form $g^R(z) = az^2$. The coefficient a=1.25 × 10^{-5} is estimated using least-square method.

A staircase linear approximation of $g^R(z)$ is then used in order to maintain formulation (3) linear. The reason for the choice of a staircase function is that, due to market rules, the aggregated bidding curve is also a stair-case function. The feasible region of R^T is split into intervals of length $I_j^r = 30$ MW $\forall j$, ranging from 0 to an upper bound of R^T chosen to be 1890 MW. For every interval, we compute the estimated marginal cost λ_j^{cap} at the mid point of the interval and set it to the height of each stair. Fig. 1 shows on the left the data points and the estimated curve of prices in Euro per MW of allocated reserve. The data appears very homoscedastic, for example, the variability around $R^T = 300$ MW is much lower than around $R^T = 450$ MW. Nevertheless, the curve is not intended to capture all the variability of the data but to represent a plausible aggregated bidding curve in the reserve market.

The cost of deploying reserves is a necessary input to Equation (3b) and must be estimated in practice, as it is unknown at the time of clearing the reserve market. We denote the marginal cost of deploying zMW of reserve by $g^r(z)$. In order to compute this cost, we approximate the clearing prices of the regulating market by a quadratic term plus an intercept, $g^r(z) = \mu + bz^2$. The parameters are estimated using the least-squares method and data relative to the clearing price of the regulating market in DK1 collected from Ref. [20]. The regulating power traded versus the market price is displayed in dots in Fig. 1. The resulting estimates of the parameters are $\mu = 48.2$ and $b = 6 \times 10^{-4}$. In order to maintain the optimization problem (3) linear, $g^r(z)$ is linearized as a stair-case function, which is shown in the right plot in Fig. 1. More complex functions could possibly be estimated, for example using time and other external factors as explanatory variables. This implementation is left for future work.

Lastly, it should be noted the difference in scale between the settlement prices of the two markets. On average, the price of allocating reserve is approximately 40 times lower than deploying them. Allocating reserve is cheaper as no energy is actually deployed but only the capacity is allocated.

4. Scenarios of reserve requirements

The total reserve capacity that should be scheduled and allocated in advance is mainly affected by three factors or uncertainty sources: the forecast error of wind power production, the forecast error of electricity demand and the forced outages of power plants, namely failures of the plants that cause their production to stop. They are all taken into account in this paper.

Suppose that wind power production is the only source of uncertainty. We assume that wind power producers bid their expected production in the day-ahead market. If the actual wind power production is greater than what was expected, then there

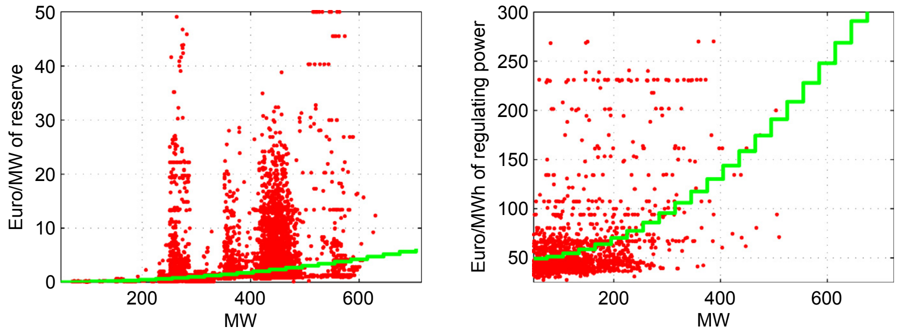

Fig. 1. On the left, the settlement price and the allocated reserves in the reserve market. Dots represent data and the staircase curve constitutes the estimated fit. On the right, the data relative to the settlement price of the regulating power market is shown in dots while the fitted stair-wise curve is displayed on top.

will be extra power to sell and hence a reduction in power supply (down-reserves) will be required to maintain the system balance; if, on the other hand, the realized wind is lower than the expected value, upward reserves will be required. In other words, if the forecasts were perfect and the errors equal to zero, no reserve would be needed. Likewise, as the forecast errors increase, more reserves are required to account for the possible mismatches between supply and demand. Similarly with the power load: it is assumed that the amount of power traded in the day-ahead energy market is equal to the expected power load demand, therefore positive errors imply upward reserve requirements while negative errors imply downward reserve requirements. The predicted outages of power plants lead directly to upward reserve requirements.

The probability distributions of the forecast errors and the power plant outages can be combined into one by convolving them, resulting in a function which will represent the probability distribution of the combined balancing requirements $f(z)$. In this paper, we draw scenarios from each individual distribution and sum them up to produce scenarios characterizing the total reserve requirements in the DK1 area of Nord Pool. A scenario-based approach is chosen because the convolution of the probability distributions of the individual stochastic variables does not have a closed form and can be highly complex. The remaining of this section elaborates on how the individual scenarios are obtained.

4.1. Scenario generation of wind power production and load forecast errors

In this subsection both the generation of scenarios of wind power production and load forecast errors are discussed. Scenarios from both stochastic variables are generated together to account for correlation between them.

Regarding the wind power production in DK1, point quantile forecast have been issued using a conditional parametric model, i.e., a linear model in which the parameters are replaced by a smooth unknown functions of one or more explanatory variables. The explanatory variables are on-line and off-line power measurements from wind turbines and numerical weather prediction of wind speed and wind direction. The functions are estimated adaptively. The errors are modeled as a sum of non-linear smooth functions of variables forecast by the meteorological model or variables derived from such forecasts. Further information about the employed modeling approach can be found in Refs. [21–23].

The load in DK1 area has been modeled as a function of the temperature, the wind, and the solar radiation. The annual trend is modeled by a cubic B-spline basis with orthogonal columns. The daily variations are modeled as a combination of different sinusoids, one referring to each time of the day. The reader is referred to [24] for a detailed description of the methodology used in this paper to model the electricity demand in DK1.

The scenarios of wind production and load are generated in pairs in order to account for their mutual correlation. Each scenario is composed by two variables and is built in three steps as in Ref. [25]: first by a sample of a multivariate Gaussian distribution where the covariance matrix is estimated recursively as new observations are collected; then, by applying the inverse probit function of such sample, and finally by using the estimated inverse cumulative function of the desired variables.

Fig. 2 shows the distribution of the scenarios of forecast errors of wind power production and load in gray and white respectively, during the 15th Dec 2011 from 13:00 to 13:59. Note that the distribution of the forecast error of wind power is wider, indicating that, in general, wind power production has a greater impact on reserve requirements than the load. On average, forecast error scenarios of wind exhibit five times more variance than the load scenarios. Finally, note that both distributions are centered around zero.

4.2. Scenario generation of power plant outages

The modeling of individual power plant outages requires historical data and specific information on each power plant which might not always be available to the TSO. Secondly, it requires computing an individual model for each unit, thus increasing complexity significantly. Thirdly, it requires information about which units will be on/off during the operation horizon, which is not available at the clearing of the Danish reserve market. An alternative approach taken in this paper is to model the total amount of MW that fail in the entire system by aggregating all the units into one. The predicted MW failed in the entire system depend on time and on the load. Historical data of power plant

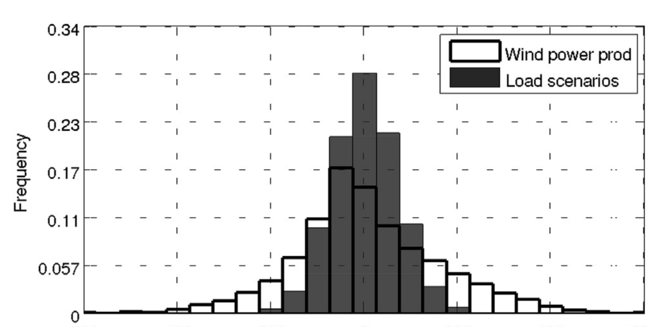

Fig. 2. The histogram of forecast error scenarios of wind power production and load during one specified hour shown are shown in gray and white, respectively.

outages can be found at the Urgent Market Messages service of Nord Pool [26]. The left plot in Fig. 3 shows the forced outages in MW during 2011. The area inside the box corresponds to the MW failed in May, also zoomed in the right plot. In the course of 2011, there was 92% of the hours where 0 MW failed; during the rest of the hours, either an outage of a single unit, a partly outage or simultaneous outages occurred.

The procedure proposed in this paper to model power plant outages is comprised of two steps. In the first step, we model the presence or absence of an outage. In the second step, we model the amount of MW failed, conditioned on the fact that a failure occurred. In Ref. [27] we explored alternative methodologies based on Hidden Markov Models that were proven to perform worse at predicting future outages.

The variable modeled in the first step Y_t is defined as.

$$Y_t = \begin{cases} 1 & \text{if failure occurs at time t} \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

It is natural to assume that Y_t follows a Bernoulli distribution, $Y_t \sim bern(p_t)$, and therefore, it is appropriate to model Y_t as a Generalized Linear Model [28]. The link function chosen is the *logit* function. The explanatory variables are the hour of the day, the day of the week and the month, all represented through sinusoidal curves. Sinusoidal terms of the form $k^{(1)}\cos(2\pi(\text{hour}_t/24))$, $k^{(2)}\cos(2\pi(\text{day}_t/7))$ and $k^{(3)}\cos(2\pi(\text{month}_t/12))$ with $k^{(1)} = 1\ldots24$, $k^{(2)} = 1\ldots7$, and $k^{(3)} = 1\ldots12$, are considered, also using the sin function. Only the most relevant were kept using a likelihood ratio test as in Ref. [28]. The final model is

$$\begin{aligned}\eta_t = \log\left(\frac{p_t}{1-p_t}\right) &= \mu + \alpha_1 cos\left(\frac{2\pi\text{day}_t}{7}\right) + \alpha_2 \sin\left(\frac{2\pi\text{day}_t}{7}\right)\\ &+ \alpha_3 \cos\left(2\frac{\pi\text{month}_t}{12}\right) + \alpha_4 \cos\left(5\frac{2\pi\text{month}_t}{12}\right)\\ &+ \alpha_5 \sin\left(\frac{2\pi\text{month}_t}{12}\right) + \alpha_6 \sin\left(2\frac{2\pi\text{month}_t}{12}\right)\\ &+ \alpha_7 \sin\left(3\frac{2\pi\text{month}_t}{12}\right) + \alpha_8 \sin\left(4\frac{2\pi\text{month}_t}{12}\right)\\ &+ \alpha_9 \sin\left(5\frac{2\pi\text{month}_t}{12}\right).\end{aligned} \tag{5}$$

The reduced model indicates that the hour of the day is not significant when predicting the probability of an outage. The day of the week and the month are both significant variables. The parameters of the model are optimized using train data and updated everyday including data from the previous 24 h during the test period.

The second stage of the model accounts for the amount of failed MW at time t, X_t, conditioned on the fact that a failure has occurred. Note that the more energy is demanded, the more power plants are online and more generators are subject to fail, meaning that the load n_t will affect our predictions of X_t. The histogram of $(X_t/n_t|Y_t = 1)$ depicted in Fig. 4 clearly resembles the density of a Gamma distribution. Thus, we assume that $(X_t/n_t|Y_t = 1) \sim \text{Gamma}(s_t, k)$, where k is the shape parameter, common for all observations, and s_t the scale parameter at time t.

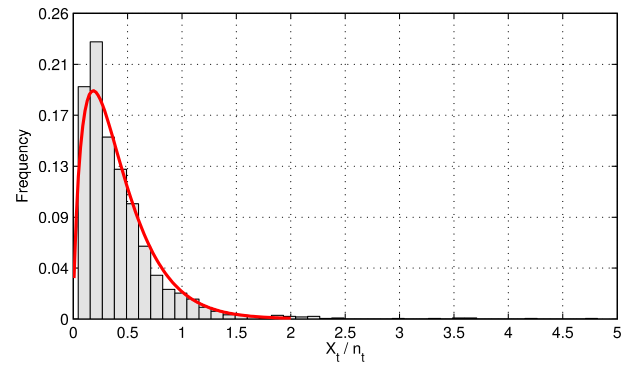

Fig. 4. Histogram of $x_t/n_t|y_t = 1$, namely the amount of MW failed divided by the load at time t, knowing that a failure has occurred. The curve represents the estimated Gamma distribution.

The probability density function of a Gamma distribution is defined as

$$f(x) = \frac{1}{\Gamma(k)s_t^k}x^{k-1}e^{-\frac{x}{s_t}}, \tag{6}$$

with mean $\mu_t = ks_t$ and variance $\sigma^2 = ks_t^2$. The canonical link for the gamma distribution is the inverse link $\eta = 1/\mu$ [28]. As in the previous binary model, the explanatory variables are several sinusoidal curves. Several approximate χ^2-distribution tests were performed to disregard irrelevant terms. The final model only including the significant terms is

$$\begin{aligned}\eta_t = \frac{1}{\mu_t} &= \mu + \alpha_1 \cos\left(\frac{2\pi\text{hour}_t}{7}\right) + \alpha_2 \cos\left(2\frac{2\pi\text{hour}_t}{7}\right)\\ &+ \alpha_3 \sin\left(\frac{\pi\text{hour}_t}{12}\right) + \alpha_4 \sin\left(2\frac{2\pi\text{hour}_t}{12}\right)\\ &+ \alpha_5 \sin\left(\frac{3\pi\text{hour}_t}{12}\right) + \alpha_6 \cos\left(\frac{2\pi\text{day}_t}{12}\right)\\ &+ \alpha_7 \cos\left(2\frac{2\pi\text{day}_t}{12}\right) + \alpha_8 \sin\left(\frac{2\pi\text{day}_t}{12}\right)\\ &+ \alpha_9 \sin\left(3\frac{2\pi\text{day}_t}{12}\right) + \alpha_{10} \cos\left(\frac{2\pi\text{month}_t}{12}\right)\\ &+ \alpha_{11} \sin\left(2\frac{2\pi\text{month}_t}{12}\right) + \alpha_{12} \sin\left(3\frac{2\pi\text{month}_t}{12}\right).\end{aligned} \tag{7}$$

When predicting the ratio X_t/n_t, the hour, the week day and the month are statistically significant.

Scenarios are generated in an iterative process. Every day at 9:00 am, the parameters of both models are updated including data from the previous day. At this time, 5,000 scenarios for each hour of the next day are generated, i.e., with lead time ranging from 16 to 40 h. Each scenario corresponds to an independent simulation of a

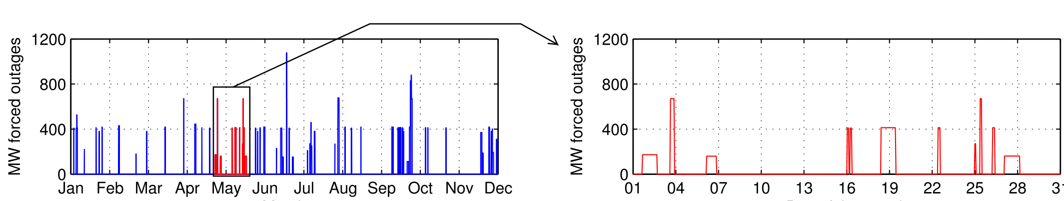

Fig. 3. Historical data of MW forced to fail in DK1. On the left, data relative to year 2011 is shown. The area inside the rectangle is zoomed on the right plot and refers to May 2011.

Bernoulli multiplied by a Gamma simulated value and by the predicted load n_t.

5. Results and discussion

The performance of the proposed reserve determination models is assessed by comparing the reserve capacity scheduled by the models and the reserve capacity actually deployed in the DK1 area of Nord Pool. The latter is calculated as the sum of the activated secondary reserve, regulating power produced in DK1 and the regulating power exchanged through the interconnections with neighboring areas. Data pertaining to the activated secondary reserve and the total volume of regulating power can be downloaded from Refs. [26] and [20], respectively. The regulating power exchanged through the interconnections with Germany, Norway, Sweden and East Denmark is estimated using data from Ref. [20]. More specifically, it is computed by subtracting the total power scheduled for each interconnector in the day-ahead market from the actual power that eventually flows through it. Primary regulation was not considered due to unavailability of the data. However, conclusions would be barely affected by considering the primary regulation as it is comparatively very low. In the remainder of the paper, a shortage event is defined as an hour when the scheduled reserves are lower than the actual reserve deployed. In practice, a shortage event does not necessarily imply that load is shed. In the Nordic market, producers who are scheduled to provide a certain capacity in the reserve market must place an offer of regulating power of the corresponding size in the regulating market, 45 min before operational time. However, other players who are not committed to provide reserves in the reserve market may still bid in the regulating market and thereby provide regulating power. This case is not considered in this paper.

The presented case study has been performed on data spanning three years. The training period of the scenario-generating models goes from the 1st January 2009 at 00:00 CET to the 30th June 2011 at 23:00 CET. The test period covers week 36 of year 2011 and weeks 3, 17 and 22 of 2012, all of them randomly selected.

Recall that scenarios are generated every day at 9:00 am with a lead time of 16–40 h, namely the next operational day. The optimization models are run using the same lead time, as if they were to be solved at the clearing of the reserve market. The solutions to the LOLP and CVaR models are compared with the actual deployed reserves in DK1 during the four testing weeks.

It is worth stressing that the proposed models for reserve determination focus on the total reserve requirements, which are triggered by unexpected fluctuations in the load and in the wind power production, and by outages of power plants. We do not distinguish, therefore, between primary, secondary and tertiary reserve. In the case of the LOLP-formulation, it is up to the TSO to decide how to split the total reserve requirements into the different types of reserve that may be considered. Likewise, the CVaR-formulation can be easily tuned to represent the three types of reserves through the estimated cost functions. Indeed, if it is much easier for plants to participate in the tertiary reserve market, because providing tertiary reserve is cheaper than providing primary and secondary reserve, then the distinction between these should be made through the supply cost function for reserve, with the tertiary reserve being cheaper than the secondary and primary ones. If, on the contrary, it is much easier for the plants to participate in the tertiary reserve market for reasons that cannot be translated into costs, then the required primary and secondary reserve should be treated as input information in the CVaR-method and subtracted from the total reserve requirements.

5.1. LOLP-model results

This subsection shows the results of applying the LOLP model introduced in Section 2.1. The model was run during the four testing weeks and the simulation results are included in Fig. 5. The actual deployed reserves in DK1 and the total scheduled reserves by Energinet.dk are also shown in the figure. The shaded areas in the background represent the solution to the LOLP model for different values of β. Weeks are separated by vertical lines. The amount of scheduled reserve varies substantially depending on the level of uncertainty of the wind, the load and the power plant outages. As an example, on the Thursday of the fourth week, the prediction of load and wind power production happened to be wrong, and up to 1,100 MW of regulating power were needed. In this special case, both the LOLP solution and Energinet.dk's reserve scheduling criterion led to a shortage event.

The reliability plot in Fig. 6 shows the desired or expected LOLP (parameter β), against the observed LOLP, namely, the number of shortage events divided by the time span. The whole data set was used to compute this reliability plot. The ideal case is illustrated by the dashed line where both quantities, expected and observed, are equal. The actual performance of the model is represented by the continuous line, which is fairly close to the ideal one, indicating that the expected probability of reserve shortage is well adjusted to the observed one.

It is worth mentioning that the value of the parameter β is directly connected to the reliability level of the underlying power system. With this in mind, a simple rule for the TSO to decide on an appropriate value for this parameter would read as follows: divide the number of hours in a year where shedding load is tolerated to happen by the total number of hours in the year. This simple rule would roughly indicate the probability that, in each hour, the need for upward balancing power exceeds the scheduled reserves. The UCTE suggests that enough reserve should be scheduled to manage energy deviations in 99,9% of all hours during the year [29]. For example, if the TSO tolerates that there are around 96 h during a year where some load might not be covered, then the resulting LOLP would be equal to 0.01. The case presented in Ref. [14] is performed using LOLP $= \{1, 0.5, 0.1\}$. The authors in Ref. [30] consider five scenarios of demand, where the LOLP is in between 0.005 and 0.016. Note, however, that the LOLP does not account for how many MW of load are shed or the cost of such load shedding events.

Next, we perform a sensitivity analysis to asses how changes in β affect the solution. We choose several plausible values of β which are displayed in the first column of Table 1, and then compute the optimal reserve schedule for each of them. The four testing weeks are considered and the results shown are averaged by the number of days. The second column shows the numbers of shortage events. The cost of allocating the reserve given by the LOLP model is displayed in the third column. It is computed using the cost function $g^R(z)$, presented in Section 3. The fourth column shows the cost of deploying the actual reserve computed using the function $g^r(z)$, which estimation is discussed in Section 3. Lastly, the MW shed, or in other words, the number of MW of actual deployed reserve exceeding the LOLP solution, is presented in the fifth column.

Note that a decrease in the parameter β implies that the solution becomes more conservative and hence, more reserve will be scheduled. For this reason, as β decreases, the number of shortage events decreases too, at the expense of increasing the allocation and deploy cost. On the other hand, the amount of MW not covered by the scheduled reserves, collated in the fourth column, decreases as β diminishes.

Next, we compare the solution to the LOLP model with the solution given by Energinet.dk. TSO's solution incurs 75 shortage

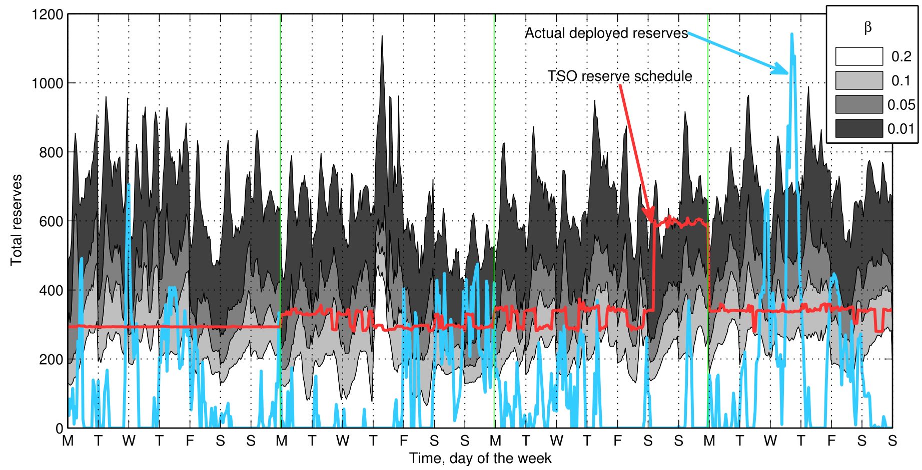

Fig. 5. The deployed reserves and the actual scheduled reserves by Energinet.dk are plotted as indicated in the text boxes. The shaded areas in the background represent the solution of the LOLP model for different values of β. Weeks are separated by vertical lines.

events during the four testing weeks, or equivalently 2.67 shortage events per day. The estimated total cost of allocating reserve is 4,355 €; the estimated deployment cost is 132,350 €, and the MW not covered 422.39.

For the same number of shortage events, the LOLP gives a worse solution that Energinet.dk's solution: the allocation costs are 3.97% higher, the deployment cost 1.18% higher and the MW shed increase in 1.15 MW per day. This means that during the four testing weeks, the LOLP methodology underperforms the solution given by Energinet.dk in terms of reliability and economic efficiency. The main advantage that the LOLP method brings is the analogy of the parameter β with the probability of a shortage event to occur, which is a very easy risk measure to interpret. On the other hand, the method has two drawbacks. As discusses before, its solution does not depend on the cost of allocating reserves, namely on λ_i or on the estimated cost function $g(z)$ (as long as it is increasingly monotonic). Neither it depends on the cost of deploying reserves. The solution only depends on the parameter β, as the relation LOLP $= \beta$ in the optimization problem (1) will always be satisfied at the optimum, no matter what the cost is. The second disadvantage is that load shedding costs are not taken into account. These drawbacks are overcome by the CVaR model, for which results are presented in the next subsection.

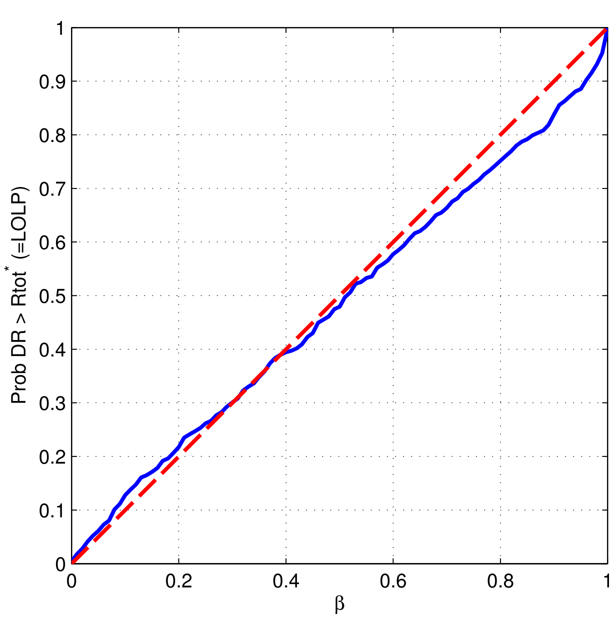

Fig. 6. Observed LOLP vs expected, namely parameter β.

5.2. CVaR-method results

In this section we discuss the results of the CVaR-based reserve determination method, which has been presented in Section 2.2, and compare them with the deployed reserves that were actually needed in DK1 during the simulation horizon.

The CVaR model needs as input two parameters which, in practice, are to be determined by the TSO: α, which controls the CVaR risk measure and represents the risk-aversion of the TSO, and V^{LOL}, which accounts for the cost in € of shedding 1 MW of load. We performed a sensitivity analysis to determine how changes in these parameters affect the level of procured reserve. The model was run for values of $\alpha = \{0, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99\}$ and $V^{LOL} = \{200, 500, 1{,}000, 2{,}000, 5\,000\}$ €/MW.

The cost of allocating and deploying reserve capacity, the cost of shedding load, and the total cost, are displayed on the upper-left, upper-right, lower-left and lower-right plot of Fig. 7. The cost is shown on the y-axis in € $\times 10^3$, while the risk parameter α is shown on the x-axis. Each line represents a cost of the reserve schedule solution for a certain V^{LOL}. All costs are averaged by the number of days in the test period. As the TSO becomes more risk averse, i.e., as α increases, the allocation and deployment costs increase, because

Table 1
The first column shows several values of parameter β which is an input to the LOLP model. The second column presents the number of shortage events on average per day. The third column includes the cost of allocating the amount of reserves given by the LOLP model on average per day. The fourth column displays the cost of deploying the actual reserve requirements. The number of MW not covered by the scheduled reserve on average per day when using the LOLP solution are shown on the fifth column.

β	Shortage events	Alloc. cost in € $\times 10^3$	Deploy. cost in € $\times 10^3$	MW not covered
0.2	5.214	1.194	95.643	862.056
0.15	4.107	2.077	110.966	668.879
0.089	2.678	4.528	134.826	423.645
0.07	1.928	6.132	143.782	346.910
0.05	1.464	9.150	153.939	270.271
0.01	0.428	33.032	187.791	85.464

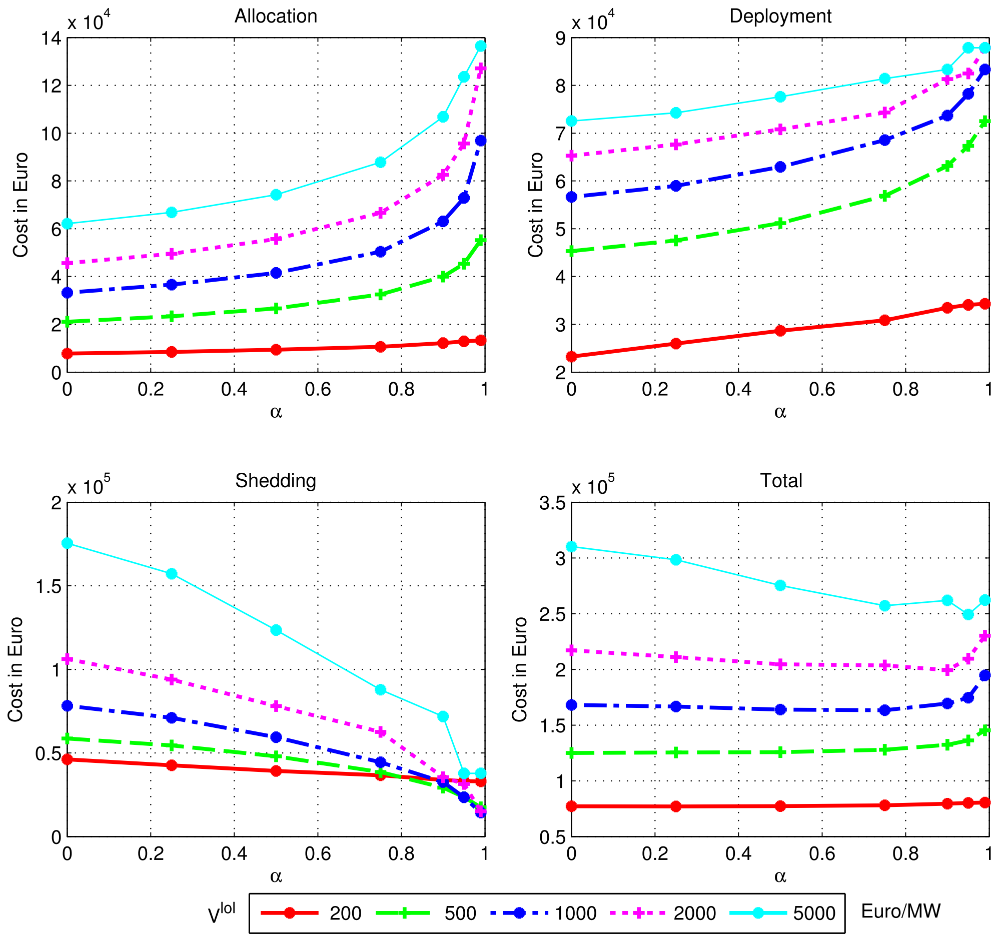

Fig. 7. Sensitivity analysis of the CVaR-based reserve determination model. The parameter α is displayed on the *x*-axis and the cost in € × 10^3 on the *y*-axis. Each line represents the cost of the solution for a certain V^{LOL}. The cost of allocating and deploying reserve capacity, the cost of shedding load and the total cost are displayed on the upper-left, upper-right, lower-left and lower-right plot, respectively. All costs are averaged by the number of days in the test period.

a larger amount of reserve is procured. The same occurs as V^{LOL} increases, since shortage events become more penalized and more reserves are scheduled to avoid them. On the other hand, the cost of curtailing load, depicted in the left-lower plot, decreases as α increases, but does not necessarily increase as V^{LOL} increases. The reason for this is that, even though the amount of curtailed load decreases as V^{LOL} increase, the product $V^{LOL} \times L_w$ representing the cost of curtailing load in scenario *w* may still increase.

The total cost shown in the down-right subplot of Fig. 7 is computed by summing up the reserve allocation, the reserve deployment and the load shedding costs. In general, the total cost increases as the risk-aversion parameter α increases. However, this is not always the case when $V^{\mathrm{LOL}} = \{1000, 2{,}000, 5{,}000\}$ €/MW. The reason for this discrepancy is that the generated scenarios do not represent the potential need for reserve capacity accurately enough. Adding more variables to the scenario representation of the reserve requirements, increasing the amount of scenarios or adding more weeks to the test period could solve this issue, in particular, they underestimate the amount of upward balancing power that may potentially be required, as confirmed by the plot in Fig. 6. Finally, one should notice that changes in the total cost are mainly driven by changes in the V^{LOL}, while changes in α have smaller impact on the solution.

The scheduled reserves when $V^{\mathrm{LOL}} = 500$ €/MW and V^{LOL}=5 000 €/MW, over time, are displayed in shadowed areas in the upper and lower plot of Fig. 8. The actual deployed reserve and the reserve scheduled by Energinet.dk are drawn on top. Weeks are separated by vertical lines. It is interesting to note how the reserve schedule given by the CVaR$_\alpha$ based reserve determination model changes as V^{LOL} changes. When minimizing the CVaR of the cost distribution of reserve allocation and deployment, and load shedding costs, an increase in V^{LOL} makes the load shedding costs have more weight in the total costs, and hence the events of reserve shortage will be penalized to a larger extent. When V^{LOL} is low, those events are less relevant and the curves look more flat. Another reason for the flatness of the curves is the linear approximation of the cost functions $g^R(z)$ and $g^r(z)$, both introduced in Section 3. The increase in cost when increasing the reserve in one unit is much higher when jumping from one step of the stepwise function to another, than when the function remains in the same step. The step lengths are defined in (3e) and in (3f). A finer linearization of such cost functions by reducing the step length would solve this issue.

The number of interruption events and the amount of load that is involuntarily shed are further analyzed in Fig. 9, in the left and right plots, respectively. As α and/or V^{LOL} increase, both the number of interruptions and the MW shed on average per day decrease. Under the assumption that only the producers committed to the reserve market are allowed to participate in the regulating market, the Danish TSO's solution incurs 75 shortage events during the four testing weeks. On average per day, the Danish TSO's solution incurs 2.67 shortage events, with an estimated total cost of allocation

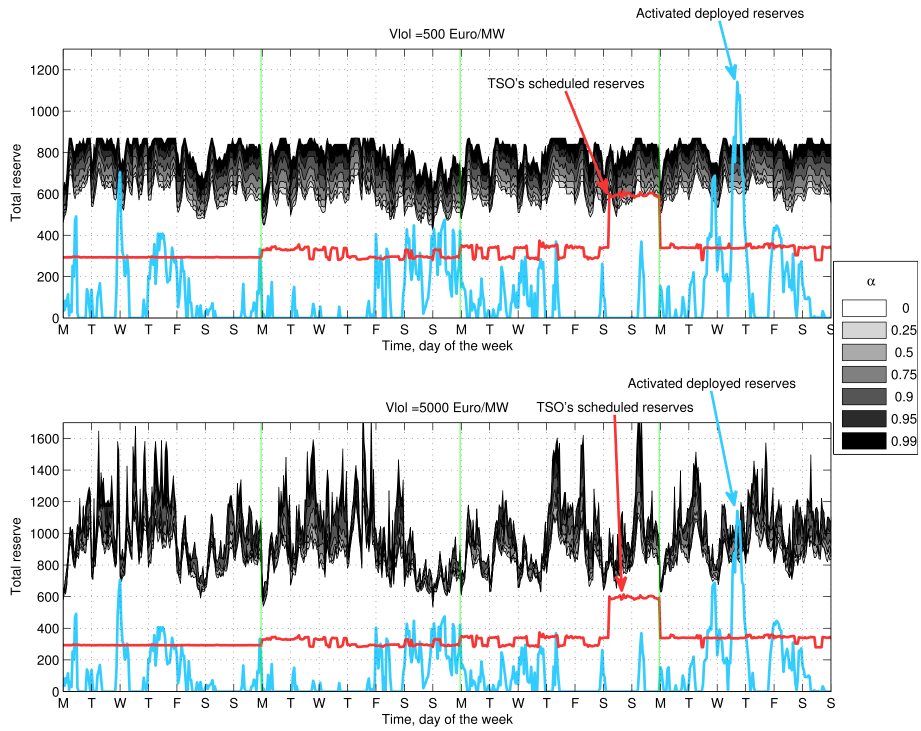

Fig. 8. The reserve schedule using the CVaR methodology is displayed for $V^{LOL} = 500$ €/MW in the upper plot and for $V^{LOL} = 5{,}000$ €/MW in the lower plot. The shaded areas in the background represent the solution of the CVaR model for different values of the risk-aversion parameter α. The actual deployed reserve in DK1 and the reserve capacity scheduled by Energinet.dk (the Danish TSO) are depicted on top.

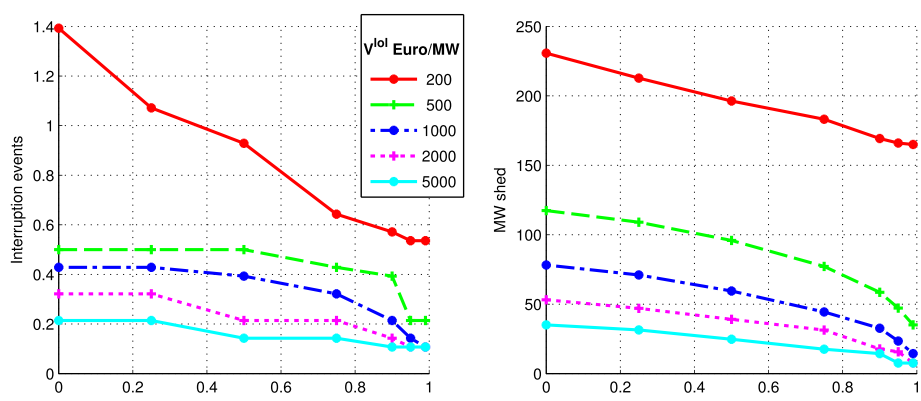

Fig. 9. In both plots, the reserve schedule, computed by the CVaR-based method, is compared to the actual deployed reserve in the DK1 area, for different values of V^{LOL} and α. On the left, we depict the average number of shortage events per day, and on the right, the average MW of load shedding.

equal to 4,355 €, an estimated deployment cost of 13,2350 €, and amount of load shed of 422.39 MW. The CVaR-method produces cheaper results in terms of total cost. The CVaR solution is from 3.38% cheaper with $\{\alpha = 0.99, V^{LOL} = 200$ €/MW$\}$, to 82.9% for $\{\alpha = 0.99, V^{LOL} = 5{,}000$ €/MW$\}$, compared to the solution given by the TSO. Note that the CVaR-method tends to schedule more reserves than the Danish TSO's solution, while at the same time the solution is cheaper, because shedding load is highly penalized by the coefficient V^{LOL}.

Fig. 10 illustrates the so-called *efficient frontier* [5]. This plot can be used by the TSO to choose an appropriate value for the risk-aversion parameter α according to its attitude towards risk. The efficiency frontier shows the expected total cost per day, namely, the expected cost of allocating and deploying reserve plus the load shedding cost per day, against the expected LOLP, that is, the expected probability of a load shedding event. The numbers along the curve indicate the value of the risk parameter α used to obtain such a point in the curve. The efficient frontier shown in Fig. 10 has been determined for a V^{LOL} equal to 5,000 €/MW. Needless to say, the efficient frontier would change for different values of V^{LOL}, but the interpretation of the resulting curves would remain similar.

The TSO can thus use this efficient frontier to resolve the trade-off between desired or expected LOLP versus the expected total cost that such a level of reliability would entail. For example, the TSO can achieve a LOLP of 0.001 with an expected total cost of approximately 3×10^5 € per day. To this end, the TSO should set the risk-aversion parameter α to 0.5. If, for instance, the LOLP is to be decreased down to 0.0002, the expected total cost would raise up to 3.2×10^5 € per day. In that case, the parameter α should be set to 0.9.

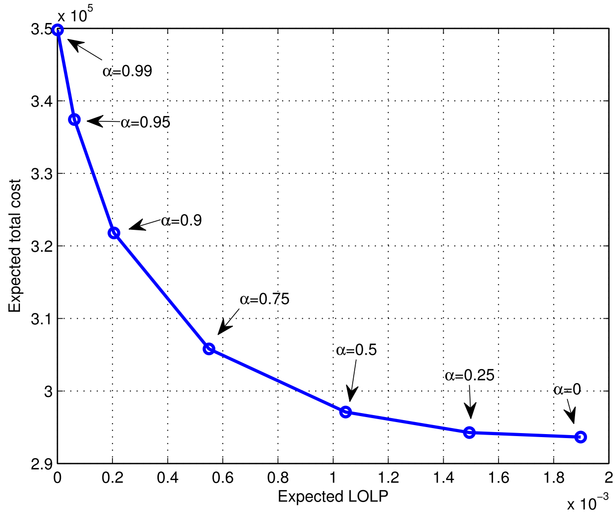

Fig. 10. Efficiency frontier plot of the CVaR model. The expected total cost, computed as the sum of the expected cost of allocating and deploying reserve, and the expected cost of shedding load for $V^{LOL} = 5{,}000$ €, is shown in the *x*-axis. In the *y*-axis, the expected LOLP is displayed. The numbers along the curve indicate the value of the risk parameter α used to obtain each of the solutions.

The main advantage that the CVaR-method offers over the LOLP method is that the TSO is able to input the cost of shedding load, V^{LOL}, in the model and, therefore, the reserve dispatch solution is dependent on it. On the contrary, the LOLP method depends only on the number of interruption events and their associated cost is not accounted for. Another advantage of the CVaR-method is that the reserve schedule depends on the reserve costs, both the allocation and deployment cost. In a real set-up, the solution would depend on the bids from the producers, while the solution of the LOLP method is independent of the reserve costs. Lastly, both optimization models are able to reflect the risk-aversion of the TSO through the risk parameters β or α. However, the risk parameter α of the CVaR methodology does not have a straightforward interpretation in real power systems, as compared to the parameter β from the LOLP-formulation, which has a direct physical interpretation.

6. Conclusion

In this paper we present two methods to determine the reserve requirements using a probabilistic approach, suited for a market structure where the reserves are scheduled independently of and before to the day-ahead energy market. This is the case in the Nordic countries and, more specifically, in the DK1 area of Nord Pool, under which the study case of this paper is framed. The first method ensures that the LOLP is kept under a certain target. The second method considers the costs of allocating and deploying reserve and of shedding load, and minimizes the CVaR of the total cost distribution at a given confidence level α. Both approaches are based on scenarios of potential balancing requirements, induced by the forecast error of the wind power production, the forecast error of the load, and the forced failures of the power plants in the power system.

The performance of the proposed reserve determination models is assessed by comparing the resulting optimal scheduled reserves with the Danish TSO's solution approach and with the actual deployed reserves during four testing weeks, in terms of costs and shortage events. The results from the case study show that the LOLP method underperforms the Danish TSO's solution in terms of costs, for the same shortage events. By using a CVaR risk approach, the cost of allocating reserves is reduced from 3.38% to 82.9%, depending on the value of the parameters of confidence level and value of lost load. The CVaR methodology provides adequate levels of reserves.

Further studies should focus on the applicability of these methods to the Nordic reserve market, by differentiating between types of reserves. This could be achieved by modeling the amount of MWh of each type of reserve required at every hour and the cost of allocating and activating each of them. Also, further improvements should be done on the modeling of the failed MW in the whole system. More specifically, time-dependencies could be modeled, since a power plant is more likely to be off-line if the previous hour was off-line too. This could be achieved by, for example, a non-homogeneous Hidden Markov Model, where the transition probabilities between states depend on time and other external variables.

Acknowledgment

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