Economic Model Predictive Control for Building Climate Control in a Smart Grid

Rasmus Halvgaard, Niels Kjølstad Poulsen, Henrik Madsen and John Bagterp Jørgensen

***Abstract*—Model Predictive Control (MPC) can be used to control a system of energy producers and consumers in a Smart Grid. In this paper, we use heat pumps for heating residential buildings with a floor heating system. We use the thermal capacity of the building to shift the energy consumption to periods with low electricity prices. In this way the heating system of the house becomes a flexible power consumer in the Smart Grid. This scenario is relevant for systems with a significant share of stochastic energy producers, e.g. wind turbines, where the ability to shift power consumption according to production is crucial. We present a model for a house with a ground source based heat pump used for supplying thermal energy to a water based floor heating system. The model is a linear state space model and the resulting controller is an Economic MPC formulated as a linear program. The model includes forecasts of both weather and electricity price. Simulation studies demonstrate the capabilities of the proposed model and algorithm. Compared to traditional operation of heat pumps with constant electricity prices, the optimized operating strategy saves 25-35% of the electricity cost.**

I. INTRODUCTION

The energy policies in the Nordic countries stipulate that 50% of the energy consumed by 2025 should come from renewable and CO_2-free energy sources. By 2050 the aim is to be independent of fossil fuels. This transformation of the energy system is needed to reduce CO_2 emissions and global warming as well as to protect the Nordic economies from the consequences of sharply rising prices of fossil fuels due to an increasing world population and depletion of fossil fuel resources. Not only the Nordic countries but the entire world and industrialized world in particular are facing this grand challenge. Reducing the fossil fuel consumption from 80% of the energy consumption to 0% in 40 years, requires introduction of a significant amount of renewable energy sources and an efficient utilization of energy in buildings, the process industries, and transportation. In the Nordic countries, a major part of the renewable energy will be produced by hydro power and offshore wind turbines. On the consumption side, residential and commercial buildings will use heat pumps for heating and electrical vehicles will replace vehicles based on combustion engines.

Accordingly, electricity will be the main energy carrier in such an energy system independent of fossil fuels. Depending on the rate of adoption of electrified vehicles, 40-70% of the energy consumption will originate from electricity in 2050. Currently, 20% of the energy consumption is electricity. As it is more difficult to store electricity than fossil fuels, such a large share of stochastic electricity production requires an intelligent power system - also referred to as a Smart Grid - that continuously balances the power consumption and the power production. This balancing requires control of the power consumption from heat pumps and electrical vehicles such that surplus of cheap wind energy is used as it is produced. Heat tanks in residential homes as well as in district heating plants must be established such that heat pumps can store electricity as heat in periods with low electricity prices. The power consumption by the process industries and retail industry, e.g. refrigeration in supermarkets and large cooling houses, must also be made flexible. Such a system is a large-scale complex system that must be coordinated to balance consumption and production of electricity.

Buildings account for approximately 40% of the total energy use in Europe. Therefore, intelligent control of the energy use in buildings is a necessity for the future smart energy system. In the Nordic countries, the energy is mainly used for heating, lighting, and electrical appliances. Heat pumps combined with water based floor heating systems will be one of the main sources for heating of buildings [1]–[4]. By themselves, these heat pumps are very energy efficient as their coefficient of performance is typically 3 or larger, i.e. for each kWh electricity supplied, they deliver more than 3 kWh heat. As heat pumps are driven by electricity and can be connected to floors with large thermal capacity, they have a large potential to shift the electricity consumption and adapt to the stochastic electricity production from wind turbines. The adoption of heat pumps could very well accelerate in the coming years. Especially for buildings situated outside district heating areas. They can benefit from heating using electric heat pumps instead of the current oil and natural gas. Heat pumps connected to the district heating system can benefit from a large store of heat and can be used to shift electricity consumption on a 24-hour or weekly basis. Furthermore, large electric heat pumps can be installed at a number of district heating plants. The large heat pumps can better exploit heat from the sea, lakes or waste heat, while small heat pumps can exploit geothermal heat.

The use of Model Predictive Control to provide indoor thermal comfort in heating systems of buildings has been reported in [5]–[7]. In the future energy systems with a large share of stochastic power producers such as wind turbines, the ability to shift the load of electricity is just as important as providing indoor thermal comfort in a heating system based on heat pumps. Different control strategies have been suggested for load balancing and load shifting in electrical grids [8]. For

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heat pumps, these strategies can be summarized as

1) **Frequency based control**. The heat pump senses the grid frequency, which in Europe has the nominal value of 50 Hz. When demand exceeds supply, the frequency falls. When supply exceeds demand, the frequency increases. Depending on a measured difference from the nominal value of the frequency, the heat pump can decide to pre-heat by starting the compressor or to delay the activation of the compressor for a short time. The advantage of this type of control is the low price of the controller, because no additional communication between the utility and the heat pump is necessary. However, there is no way to integrate the device into an intended schedule as it responds completely autonomously.
2) **Price based control**. The heat pump controller computes a schedule for the compressor based on dynamic price information given by the utility. This enables the heat pump to shift its load to times with low electricity price. It requires a communication infrastructure between utilities and households. The drawback of this control strategy is that it is relatively complex and the fact that effects of sent tariff information to affect the load are not completely sure for the utility.
3) **Direct control**. Given the communication infrastructure required for the price based control, utilities can send control signals to the heat pump to raise or reduce the demand. This allows the utility a more direct control of the demand. Furthermore, it allows the controller in the heat pump to be quite simple as it only sends information and receives commands from the utility. The drawback of course is that the utility must solve large-scale optimization problems to coordinate a large number of heat pumps.

[9] use Economic Model Predictive Control (MPC) in a direct control case to shift the electricity load of refrigeration systems. In this paper, we use Economic MPC based on price signals to control a heat pump such that certain temperature limits in a building with a floor heating system are respected. By using price signals, both current and future prices, the optimization of the energy consumption of each individual residential building decouples from the energy consumption of all other agents in the system. However, we do not specify how to determine this price but assume that it is given based on market principles of supply and demand. Consequently, each individual house is a price taker.

Simple weather conditions such as outdoor temperature and solar radiation are included in the model. By adding forecasts of prices and weather conditions to the heat pump control problem, the energy consumption is made flexible. It is thus possible to predict where to place the heat pump energy consumption and minimize the electricity cost of operating the heat pump to meet a certain indoor thermal comfort, i.e. a desired temperature interval. The temperature interval can be time varying. We exploit that the dynamics of the temperatures in the house floor heating system and indoor air are slow while the power consumption can be changed rapidly. The thermal capacity of the residential building determines how much of the electricity consumption that can be shifted to times with cheap electricity.

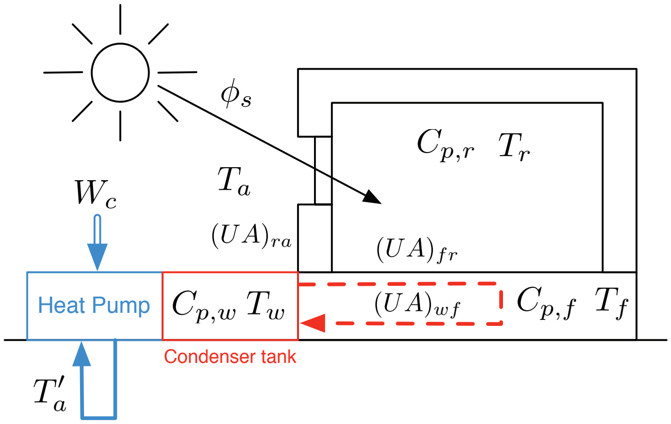

Fig. 1. House and heat pump floor heating system and its thermal properties. The dashed line represents the floor heating pipes.

MPC is increasingly being considered for building climate control [10]–[12]. Traditionally, MPC is designed using objective functions penalizing deviations from a given set-point. Recently Economic MPC has emerged as a general methodology with efficient numerical implementations and provable stability properties [13], [14].

This paper is organized as follows. In Section II, we develop and discuss a model for a heat pump connected to a floor heating system of a building. Section III states and discusses the Economic MPC. Simulation results for the Economic MPC applied to the model are described in Section IV. Section V provides conclusions.

II. MODEL

In this section, we develop a model of the heat dynamics of a house floor heating system connected to a ground source based heat pump. The system is illustrated in Fig. 1. The model is a linear third order model. Table I lists the variables and parameters of the model.

TABLE I
DESCRIPTION OF VARIABLES

Variable	Unit	Description
T_r	°C	Room air temperature
T_f	°C	Floor temperature
T_w	°C	Water temperature in floor heating pipes
T_a	°C	Ambient temperature
T'_a	°C	Ground temperature
W_c	W	Heat pump compressor input power
ϕ_s	W	Solar radiation power

A. Energy Balances and Heat Conduction

In this subsection we develop energy balances for the air in the room, the floor and the water in the floor heating pipes and condenser water tank. In the simple model developed in this paper, the house is considered to be one big room. Furthermore, we make the following simplifying assumptions: 1) One uniform air temperature, 2) no ventilation, 3) no influence from humidity of the air, 4) no influence from the heat released from people in the room, 5) no influence from wind.

In [15] a model of the indoor temperature in buildings is identified and suggests at least two dominating heat accumulating media in order to capture the short-term and long-term variations of the heat dynamics. In our model two heat accumulating media are thus included, namely the room air and the floor. The resulting energy balances are

$$C_{p,r}\dot{T}_r = Q_{fr} - Q_{ra} + (1-p)\phi_s \tag{1}$$

$$C_{p,f}\dot{T}_f = Q_{wf} - Q_{fr} + p\,\phi_s \tag{2}$$

The disturbances are the ambient temperature and the solar radiation through a window. These disturbances are also illustrated in Fig. 1.

The energy balance for the water circulating in the floor heating pipes can be stated as

$$C_{p,w}\dot{T}_w = Q_c - Q_{wf} \tag{3}$$

in which Q_c is the heat transferred to the water from the condenser in the heat pump. Q_{wf} is the heat transferred from the water to the floor.

The conductive heat transfer rates are

$$Q_{ra} = (UA)_{ra}(T_r - T_a) \tag{4a}$$

$$Q_{fr} = (UA)_{fr}(T_f - T_r) \tag{4b}$$

$$Q_{wf} = (UA)_{wf}(T_w - T_f) \tag{4c}$$

Q_{ra} is the heat transferred from the air in the room to the surroundings, Q_{fr} is the heat transferred from the floor to the air in the room, and Q_{wf} is the heat transferred from the water in the floor heating pipes to the floor. The term $U \cdot A$ is a product of the heat conductivity and the surface area of the layer between two heat exchanging media. Its reciprocal value $R = 1/(UA)$ is often used since it can be interpreted as a resistance against heat flow [16].

B. Heat Pump

A heat pump is a device that transfers heat from a low temperature zone to a higher temperature zone using mechanical work. A heat pump can provide both heating or cooling, but in cooler climates heating is of course more common. Heat pumps normally draw heat from the air or from the ground and uses a vapor compression refrigeration cycle. This cycle requires the four basic components as sketched in Fig. 2. The components are a compressor, an expansion valve, a condenser converting the working fluid from its gaseous state to its liquid state, and an evaporator converting the working fluid from its liquid state to its gaseous state [17], [18].

As the heat pump dynamics is much faster than the thermodynamics of the building, we can assume a static model for the heat pump. The amount of heat transferred from the condenser to the water, Q_c, is related to the work of the compressor, W_c, using the coefficient of performance

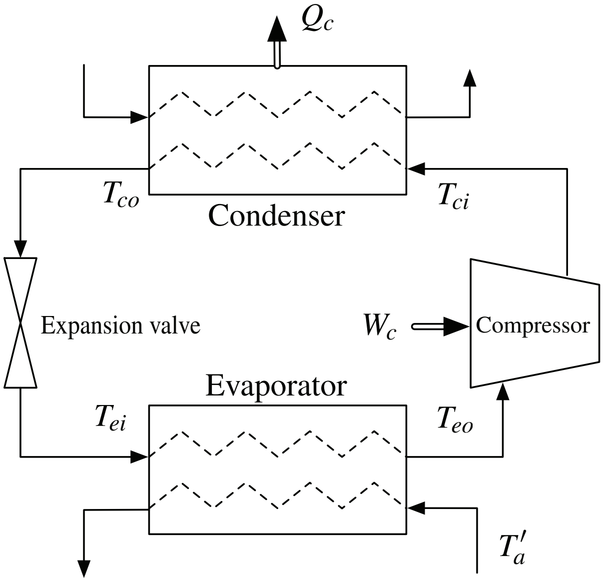

Fig. 2. Heat pump vapor compression refrigeration cycle. The temperatures are denoted T with subscript c or e for condenser or evaporator, respectively, while i or o denotes input or output.

$$Q_c = \eta W_c \tag{5}$$

The coefficient of performance η for heat pumps varies with type, outdoor ground temperature, and the condenser temperature. As the outdoor ground temperature and the condenser temperature are approximately constant, we can assume that the coefficient of performance is constant. For ground source based heat pumps η is typically around 3 in the specified operating range.

The model consists of (1)-(5). Consequently, a third order linear model can be stated as

$$\begin{aligned} C_{p,r}\dot{T}_r = (UA)_{fr}(T_f - T_r)\ldots \\ - (UA)_{ra}(T_r - T_a) + (1-p)\phi_s \end{aligned} \tag{6a}$$

$$\begin{aligned} C_{p,f}\dot{T}_f = (UA)_{wf}(T_w - T_f)\ldots \\ - (UA)_{fr}(T_f - T_r) + p\phi_s \end{aligned} \tag{6b}$$

$$C_{p,w}\dot{T}_w = \eta W_c - (UA)_{wf}(T_w - T_f) \tag{6c}$$

C. State space model

The model (6) can be expressed as a continuous-time state space model

$$\dot{x} = Ax + Bu + Ed \tag{7a}$$

$$y = Cx \tag{7b}$$

x is the states, u is the manipulated variables, d is the disturbances, and y is the controlled variable. In the case studied, the states are $x = \begin{bmatrix} T_r & T_f & T_w \end{bmatrix}^T$; the manipulate variable is the power used by the compressor in the heat pump, $u = W_c$; the disturbances are the ambient temperature and the sun radiation such that $d = \begin{bmatrix} T_a & \phi_s \end{bmatrix}^T$; and the controlled

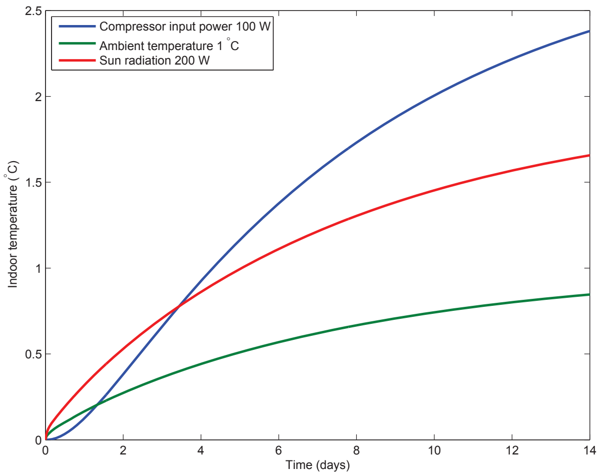

Fig. 3. Step responses from inputs and disturbances to indoor temperature T_r. Step size is noted in the plot legend.

variable is the indoor temperature $y = T_r$. The matrices in the state space model (7) are

$$A = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix} \qquad E = \begin{bmatrix} \frac{(UA)_{ra}}{C_{p,r}} & \frac{1-p}{C_{p,r}} \\ 0 & \frac{p}{C_{p,f}} \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 & \frac{\eta}{C_{p,w}} \end{bmatrix}^T$$

with the coefficients

$$\begin{aligned} a_{11} &= (-(UA)_{fr} - (UA)_{ra})/C_{p,r} & & \\ a_{22} &= (-(UA)_{wf} - (UA)_{fr})/C_{p,f} & a_{33} &= -(UA)_{wf}/C_{p,w} \\ a_{12} &= (UA)_{fr}/C_{p,r} & a_{23} &= (UA)_{wf}/C_{p,f} \\ a_{21} &= (UA)_{fr}/C_{p,f} & a_{32} &= (UA)_{wf}/C_{p,w} \end{aligned}$$

[19] provides values for building heat capacities and thermal conductivities obtained from system identification methods. The values of these parameters for a representative building are listed in Table II. The water tank heat capacity is estimated as $C_{p,w} = m_w c_w$ for a 200 liter tank filled with water having the specific heat capacity c_w and mass m_w. The resulting time constants of the third order model are 1, 24, and 186 hours for the room air, water condenser tank, and the floor, respectively. This is also observed from the step responses seen in Fig. 3.

III. ECONOMIC MPC

The state space model (7) is converted to a discrete-time state space model using zero-order-hold sampling of the input signals

$$x_{k+1} = A_d x_k + B_d u_k + E_d d_k \tag{8a}$$

$$y_k = C_d x_k \tag{8b}$$

Using this discrete-time linear state space formulation to predict the future outputs, we may formulate a linear program that minimizes the electricity cost for operating the heat pump while keeping the indoor room temperature in prespecified intervals

$$\min_{\{x,u,y\}} \quad \phi = \sum_{k \in \mathcal{N}} c_{u,k} u_k + \rho_v v_k \tag{9a}$$

$$\begin{aligned} s.t. \quad & x_{k+1} = A_d x_k + B_d u_k + E_d d_k & k \in \mathcal{N} \quad & (9b) \\ & y_k = C_d x_k & k \in \mathcal{N} \quad & (9c) \\ & u_{\min} \leq u_k \leq u_{\max} & k \in \mathcal{N} \quad & (9d) \\ & \Delta u_{\min} \leq \Delta u_k \leq \Delta u_{\max} & k \in \mathcal{N} \quad & (9e) \\ & y_{k,\min} \leq y_k + v_k & k \in \mathcal{N} \quad & (9f) \\ & y_{k,\max} \geq y_k - v_k & k \in \mathcal{N} \quad & (9g) \\ & v_k \geq 0 & k \in \mathcal{N} \quad & (9h) \end{aligned}$$

$\mathcal{N} \in \{0, 1, \dots, N\}$ and N is the prediction horizon. The electricity prices enter the optimization problem as the cost coefficients $c_{u,k}$. It may not always be possible to meet the temperature demand. Therefore, the MPC problem is relaxed by introduction of slack variable v_k and the associated penalty cost ρ_v. The penalties can be set sufficiently large, such that the output constraints are met whenever possible. The Economic MPC also contains bound constraints and rate-of-movement constraints on the manipulated variables. The rate-of-movement is defined in discrete time as $\Delta u_k = u_{k+1} - u_k$ and adds to robustness of the numerical optimization routine.

The prediction horizon, N, is normally selected large to avoid discrepancies between open-loop and closed-loop profiles. However, long horizons increases computation speed rapidly and uncertainties in the forecasts grow larger and larger with time. At each sampling time, we solve the linear program (9) to obtain $\{u_k^*\}_{k=0}^{N-1}$. We implement u_0^* on the process. As new information becomes available at the next sampling time, we redo the process of solving the linear program using a moving horizon and implementing the first part, u_0^*, of the solution.

The electricity prices, $\{c_{u,k}\}_{k=0}^{N-1}$, as well as the ambient temperature and sun radiation, $\{d_k\}_{k=0}^{N-1}$, must be forecasted. In this paper we assume that we have perfect forecasts.

IV. RESULTS

The Economic MPC has been implemented in Matlab calling a primal active set solver. To illustrate the potential of the Economic MPC for controlling heat pumps, we simulate scenarios using day-ahead electricity prices from Nordpool, the Nordic power exchange market. These electricity prices are available in one hour intervals. We also discretize the system using a sample time of 30 minutes, i.e. $T_s = 0.5$ hour. Both the outdoor temperature, T_a, and solar radiation ϕ_s are modeled as diurnal cycles with added noise [10]. We aim to minimize the total electricity cost in a given period while keeping the indoor temperature, T_r, in predefined intervals. In the case studied, we assume that the forecasts are perfect, i.e. that the forecasts are without uncertainty. We simulate a five day period using a prediction horizon $N = 96$ (= 48 hours). The optimal control signal is calculated at every time step over the prediction horizon to obtain a closed loop profile.

Fig. 4 illustrates the optimal compressor schedule and the predicted indoor temperature for a five day horizon. The lower

TABLE II
ESTIMATED MODEL PARAMETERS

	Value	Unit	Description
$C_{p,r}$	810	kJ/°C	Heat capacity of room air
$C_{p,f}$	3315	kJ/°C	Heat capacity of floor
$C_{p,w}$	836	kJ/°C	Heat capacity of water in floor heating pipes
$(UA)_{ra}$	28	kJ/(°C h)	Heat transfer coefficient between room air and ambient
$(UA)_{fr}$	624	kJ/(°C h)	Heat transfer coefficient between floor and room air
$(UA)_{wf}$	28	kJ/(°C h)	Heat transfer coefficient between water and floor
c_w	4.181	kJ/(°C kg)	Specific heat capacity of water
m_w	200	kg	Mass of water in floor heating system
p	0.1		Fraction of incident solar radiation on floor
η	3		Compressor coefficient of performance (COP)
ρ_v	10^4		Slack variable penalty

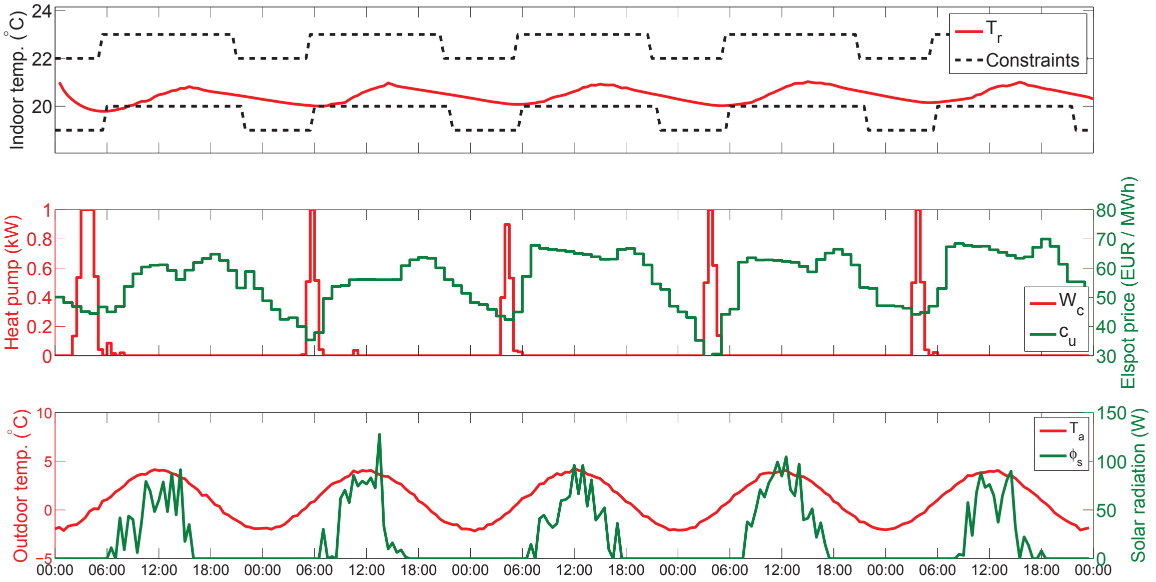

Fig. 4. Temperature in a house with time varying soft constraints, time varying electricity prices, and time varying outdoor temperatures. The simulation time is five days starting 20 JAN 2011 00:00. The upper figure shows the indoor temperature, the middle figure contains the electricity spot price and the optimal schedule for the heat pump, and the lower figure contains the ambient temperature and solar radiation. The compressor is on when the electricity spot price is low.

plot shows the outdoor temperature, T_a, and the solar radiation, ϕ_s. The outdoor temperature reflects a cold climate, i.e. the outdoor temperature is lower than the indoor temperature. The solar radiation has a peak around noon contributing to heating the building. The middle plot shows the actual electricity prices in Western Denmark. The middle plot also contains the computed optimal heat pump power input, W_c. The upper plot shows the predicted indoor temperature along with the predefined time varying constraints. The constraints indicate that during night time the temperature is allowed to be lower than at day time. The figure reveals clearly that the power consumption is moved to periods with cheap electricity and that the thermal capacity of the house floor is able to store enough energy such that the heat pump can be left off during day time. This demonstrates that the slow heat dynamics of the floor can be used to shift the energy consumption to periods with low electricity prices and still maintain acceptable indoor temperatures. Notice that the soft constraints are violated in the beginning due to the initial conditions. We allow for such violations by using reasonable moderate penalty costs for violation of the soft constraints. Consequently, the controller will find cheaper optimal solutions while the comfort level is compromized very little.

We also conducted a simulation with constant electricity prices. In this case, the heat pump now is turned on to just keep the indoor temperature at its lower limit. This implies that there is no load shifting from the heat pump in this case. By comparing the case with varying electricity price, $\{u_k^*\}_{k=0}^{N-1}$, to the case with constant electricity price, $\{u_{k,cst}^*\}_{k=0}^{N-1}$, we observe economic savings around 35%. We obtained this figure

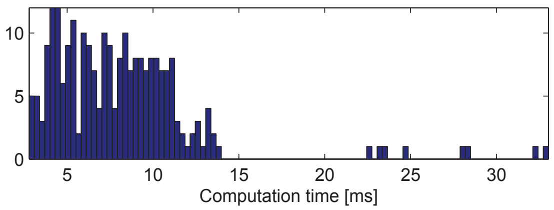

Fig. 5. Computation time distribution for all open loop profiles calculated in the five days closed loop simulation with prediction horizon 48 hours.

by comparing the total electricity expenses using the true time varying electricity prices $\{c_{u,k}\}_{k=0}^{N-1}$ such that the savings S are calculated as

$$S = -\frac{c_u^T u_{cst}^* - c_u^T u^*}{c_u^T u_{cst}^*} \tag{10}$$

Using a simulation study with hard constraints on the indoor temperature, the saving by load shifting was 25%.

Figure 5 shows the computation times of solving the open loop optimization problems for the given simulation using a PC with Intel Core i7 2.67 GHz. The average computation time is seen to be around 8 ms. Using hard constraints the average computation time reduces to 1 ms.

V. CONCLUSIONS

In this paper, we have presented a model for the temperature in a residential building with a floor heating system and a heat pump. We used an Economic Model Predictive Controller (Economic MPC) to manipulate the compressor in the heat pump such that the total electricity cost is minimized, while keeping the indoor temperature in a predefined interval. Using actual electricity prices and weather conditions, we demonstrated that the Economic MPC is able to shift the power consumption load to periods with low electricity prices. As the Nordic Electricity spot prices reflect the amount of wind power in the system, the large thermal capacity of the house floor can essentially be used to store cheap electricity from renewable energy sources such as wind turbines. We also observed that the load shifting ability of the Economic MPC can exploit weather forecasts to reduce the total cost of operating a heat pump.

The Economic MPC concept was proofed using perfect forecasts. In the future, we will use real forecast to investigate cases with uncertainty.

REFERENCES

[1] A. Hepbasli and Y. Kalinci, “A review of heat pump water heating systems,” *Renewable and Sustainable Energy Reviews*, vol. 13, pp. 1211–1229, 2009.

[2] F. Karlsson and P. Fahlén, “Impact of design and thermal intertia on the energy saving potential of capacity controlled heat pump heating systems,” *International Journal of Refrigeration*, vol. 31, pp. 1094–1103, 2008.

[3] Z. Han, M. Zheng, F. Kong, F. Wang, Z. Li, and T. Bai, “Numerical simulation of solar assisted ground-source heat pump heating system with latent heat energy storage in severly cold area,” *Applied Thermal Engineering*, vol. 28, pp. 1427–1436, 2008.

[4] A. Molyneaux, G. Leyland, and D. Favrat, “Environomic multi-objective optimisation of a district heating network considering centralized and decentralized heat pumps,” *Energy*, vol. 35, pp. 751–758, 2010.

[5] T. Y. Chen, “Application of adaptive predictive control to a floor heating system with a large thermal lag,” *Energy and Buildings*, vol. 34, pp. 45–51, 2001.

[6] H. Karlsson and C.-E. Hagentoft, “Application of model based predictive control for water-based floor heating in low energy residential buildings,” *Building and Environment*, vol. 46, pp. 556–569, 2011.

[7] S. Privara, J. Siroky, L. Ferkl, and J. Cigler, “Model predictive control of a building heating system: The first experience,” *Energy and Buildings*, vol. 43, pp. 564–572, 2011.

[8] M. Stadler, W. Krause, M. Sonnenschein, and U. Vogel, “Modelling and evaluation of control schemes for enhancing load shift of electricity demand for cooling devices,” *Environmental Modelling & Software*, vol. 24, pp. 285–295, 2009.

[9] T. G. Hovgaard, K. Edlund, and J. B. Jørgensen, “The potential of economic MPC for power management,” in *49th IEEE Conference on Decision and Control*, 2010, pp. 7533–7538.

[10] F. Oldewurtel, A. Ulbig, A. Parisio, G. Andersson, and M. Morari, “Reducing peak electricity demand in building climate control using real-time pricing and model predictive control,” *IEEE Conference on Decision and Control*, 2010.

[11] F. Oldewurtel, A. Parisio, C. N. Jones, M. Morari, D. Gyalistras, M. Gwerder, V. Stauch, B. Lehmann, and K. Wirth, “Energy efficient building climate control using stochastic model predictive control and weather predictions,” *Proceedings of ACC*, 2010.

[12] V. M. Zavala, E. M. Constantinescu, and T. K. amd Mihai Anitescu, “On-line economic optimization of energy systems using weather forecast information,” *Journal of Process Control*, no. 19, pp. 1725–1736, 2009.

[13] J. B. Rawlings and R. Amrit, “Optimizing process economic performance using model predictive control,” in *Nonlinear Model Predictive Control: Towards New Challenging Applications*. Springer, 2009, pp. 119–138.

[14] M. Diehl, R. Amrit, and J. B. Rawlings, “A lyapunov function for economic optimizing model predictive control,” *IEEE Transactions on Automatic Control*, 2009.

[15] H. Madsen and J. Holst, “Estimation of continuous-time models for the heat dynamics of a building,” *Energy and Buildings*, vol. 22, no. 1, pp. 67–79, 1995.

[16] A. Thavlov, “Dynamic optimization of power consumption,” Master’s thesis, Dept. of Informatics and Mathematical Modelling, Technical University of Denmark, 2008.

[17] A. Schijndel and M. de Wit, “Advanced simulation of building systems and control with simulink,” *Building Simulation*, pp. 1185–1192, 2003.

[18] B. P. Rasmussen, “Dynamic modeling and advanced control of air conditioning and refrigeration systems,” Ph.D. dissertation, University of Illinois, 2000.

[19] K. K. Andersen, H. Madsen, and L. H. Hansen, “Modelling the heat dynamics of a building using stochastic differential equations,” *Energy and Buildings*, vol. 13, pp. 13–24, 2000.