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Identification of a Linear Continuous Time Stochastic Model of the Heat Dynamics of a Greenhouse

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In order to use a model-based optimal control strategy for the heat supply to greenhouses, a reasonable description of the heat dynamics is required. This paper describes the identification of a linear stochastic model for the heat dynamics of a greenhouse which takes the global radiation, the outdoor air temperature and the heat supply as input variables. The model is a linear and lumped parameter model formulated in state-space form in continuous time. The formulation contains physically interpretable parameters and, for their identification, data from an experiment conducted in winter have been used. During the experiment, the heat supply was controlled by a pseudo-random binary signal (PRBS) in order to avoid a correlation between the heat supply and other variables, and in order to ensure that the dynamic characteristics of the greenhouse were present in the data.

A number of alternative model structures have been considered, and by using statistical methods a model with three thermal capacities is suggested. This model predicts the air temperature 2 min ahead with a standard deviation of 0·062 K.

Physical knowledge as well as statistical methods are used to validate the model. The estimated parameters of the model show reasonable agreement with prior physical knowledge. Additionally, the model has been verified by simulating the air temperature using an independent set of data.

Notation

A_s	effective horizontal glass area exposed to the global radiation/m^2
C_j	heat capacity of node *j*, J/K
$e(t)$	measurement error of the air temperature in the greenhouse, K
N	number of observations
p	number of parameters in a model
Q_h	energy (heat) input from the heating system, W
q_{in}	input energy, J
q_{out}	output energy, J
q_{st}	stored energy, J
R_0	resistance to heat transfer between indoor and outdoor air, K/W
R_j	resistance to heat transfer between nodes j and $j+1$, K/W
T_0	temperature of the outdoor air, K
T_j	temperature of node *j*, K
T_r	measured air temperature in the greenhouse, K
σ_{jj}^2	variance of the temperature T_j in note *j*
σ_1^2	variance of the measurement error of the air temperature
ϕ_s	heat flux from solar radiation, W/m^2

1. Introduction

The purpose of this paper is to describe the basis for improving the control of air temperature and heat supply in greenhouses using a method which controls the energy supply by a model-based prediction of the air temperature in the greenhouse. Controllers of this type are the minimum variance controller,[1] the generalized predictive controller[2] and the proportional-integral-plus (PIP) controller.[3] Prediction-based controllers have proved to be powerful in controlling the supply temperature in a distinct heating system.[4] Model-based adaptive control of greenhouses has previously been considered by Udink ten Cate,[5] Davis and Hooper,[6] Young *et al*.,[7,8] Sigrimis and Rerras[9] and Nielsen and Madsen.[10] In the control model used by Udink ten Cate,[5] the heat transfer is described by a first-order differential equation. Inputs to the model are solar radiation, long-wave sky radiation, external air temperature, wind speed, heating and ventilation. Davis and Hooper[6] use traditional time-series analysis to identify models where the internal air temperature is described by three inputs, each with a first-order transfer function. The inputs are the pipe temperature of the heating system, external temperature and wind speed. In general, the performance of prediction-based control systems depends on the possibility of obtaining good predictions of the air temperature in the greenhouse in response to the heating control input.

A total model of the heat dynamics of greenhouses includes equations of heat transfer by diffusion and radiation, flow of air, evaporation and condensation of water and such a model becomes a very complex system of equation.[11,12] Due to its complexity, a total model of the heat dynamics is unsuitable for a predictive control system design. Therefore, there is a need for identifing smaller models that describe the significant variations in the heat dynamics. Such models may be identified by stochastic modelling.[6–8,13] However, in horticultural engineering there is a profound need to formulate stochastic continuous-time models of heat dynamics of greenhouses where knowledge about the physics of heat dynamics may be included in the model so that the estimated parameters are given a physical expression. Thus, this paper describes how to identify an approximate model of the heat dynamics in a greenhouse which describes the essential dynamics by parameters of physical interpretation and, using the heat supply, the global radiation and the outdoor air temperature as input variables. The model predicts the air temperature by a number of coupled linear differential equations and describes the deviations between measurement and predicted air temperature by a stochastic component.

2. The heat dynamics in a greenhouse

Heat transfer is energy in transit due to a temperature difference between two nodes with different temperatures. Considering a whole greenhouse, it becomes a tedious task to calculate the heat diffusion equation for heat conduction in all parts of the house and simultaneously calculate the heat transfer between the air and all the surfaces of glass, ground, plants, etc., by the equations for convection and radiation. Furthermore, such a detailed calculation seems to be meaningless in practice due to the uncertain specification of the heat capacities and conductivities. Therefore, the heat dynamics in the greenhouse have to be simplified. A simple, yet common, simplification is the lumped capacitance method.[14] The essence of this method is the assumption that the heat capacities of the greenhouse are lumped in certain nodes where the temperature in each node is spatially uniform.

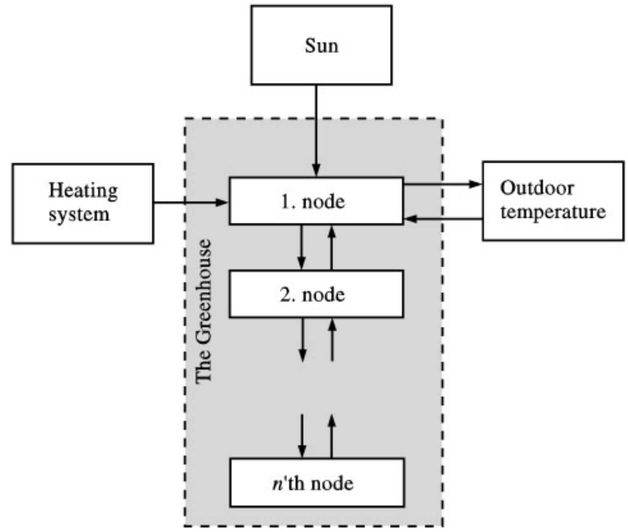

Fig. 1. Serial energy fluxes in a greenhouse with n nodes. Each node is assumed to be spatially uniform with a constant temperature and heat capacity

For such a model, the energy balance at each node in the greenhouse is

$$\frac{dq_{st}}{dt} = \frac{dq_{in}}{dt} - \frac{dq_{out}}{dt} \quad (1)$$

where q_{st}, q_{in} and q_{out} (in J) are the stored, the input and the output energy, respectively, for the node.

Figure 1 shows a simple model of the heat dynamics in a greenhouse. Since the heat input from solar radiation has a dominant immediate effect on the internal conditions, the solar radiation is considered as the input to the first node. In each node of *Fig. 1* the change of energy is represented by the change of temperature, i.e.

$$C_j \frac{dT_j}{dt} = \frac{dq_{in,j}}{dt} - \frac{dq_{out,j}}{dt} \quad (2)$$

where T_j (in K) is the temperature and C_j (in J/K) is the heat capacity of the jth node. Thus, the energy balance of the first node in *Fig. 1* is

$$C_1 \frac{dT_1}{dt} = \frac{T_2 - T_1}{R_1} + \frac{T_0 - T_1}{R_0} + Q_h + A_s\phi_s \quad (3)$$

where T_1, T_2 and T_0 are the temperatures of nodes 1, 2 and the outdoor air, respectively; R_0 (in K/W) is the resistance to heat transfer between the inside and the outside; R_1 (in K/W) is the resistance to heat transfer between nodes 1 and 2. The energy Q_h (in W) is the heat input from the heating system; ϕ_s (in W/m^2) is the heat flux from solar radiation; and A_s (in m^2) describes the effective horizontal glass area exposed to the global radiation, i.e. the area corrected for reflection, shading, dust, etc. The parameter R_0 contains a description of the mean influence of the wind speed. However, to describe R_0 as a function of the wind speed makes the model non-linear and the identification becomes much more difficult.

The energy balance of the jth node ($j \geqslant 1$) is determined approximately by

$$C_j \frac{dT_j}{dt} = \frac{T_{j-1} - T_j}{R_{j-1}} + \frac{T_{j+1} - T_j}{R_j} \tag{4}$$

where R_j (in K/W) is the resistance to heat transfer between nodes j and $j+1$. For the last node $j = n$, the second term with $j+1$ of Eqn (4) is eliminated, since heat flux from the lower nodes is assumed to be negligible. Modelling the heat transfer between two nodes by Eqn (4) is reasonable owing to the one-dimensional heat diffusion through a homogeneous layer and the convective heat transfer. The heat transfer due to radiation between two nodes is determined by the non-linearity in the heat flux. However, at temperatures from about 270 to 300 K, the linear approximation in Eqn (4) includes a significant part of the energy transfer by radiation. In summary, the resistance to heat transfer R_j between two nodes will be a weighted mean of the one-dimensional heat diffusion, the convection and the linear approximation of the radiation.

From Eqns (3) and (4), it is seen that the lumped capacitance method describes the heat dynamics in a greenhouse by a number of coupled first-order differential equations. After dividing each differential equation by the corresponding heat capacity, C_j, Eqns (3) and (4) can be written in the matrix notation as a linear differential equation

$$\frac{d}{dt}\mathbf{T} = \mathbf{AT} + \mathbf{BU} \tag{5}$$

where

$$\mathbf{T} = \begin{pmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} -\frac{1}{C_1R_0} - \frac{1}{C_1R_1} & \frac{1}{C_1R_1} & 0 & \vdots & \vdots & 0 \\ \frac{1}{C_2R_1} & -\frac{1}{C_2R_1} - \frac{1}{C_2R_2} & \frac{1}{C_2R_2} & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \frac{1}{C_nR_{n-1}} & -\frac{1}{C_nR_{n-1}} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} \frac{1}{C_1R_0} & \frac{1}{C_1} & \frac{A_s}{C_1} \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{U} = \begin{pmatrix} T_0 \\ Q_h \\ \phi_s \end{pmatrix}$$

Since the proposed model describes the heat dynamics by a number of coupled differential equations, it is well suited for the prediction of change in the temperatures at the nodes.

3. A linear stochastic differential equation

In order to describe the deviation between Eqn (5) and the actual temperatures, an additive noise process is introduced. Then the model of the heat dynamics is described by the linear stochastic differential equation

$$dT = \mathbf{AT}\,dt + \mathbf{BU}\,dt + d\boldsymbol{w}(t) \tag{6}$$

where $d\boldsymbol{w}(t)$ is assumed to be a stochastic process with independent increments. In order to estimate the parameters in the model by the maximum likelihood method, $\boldsymbol{w}(t)$ is assumed to be a Wiener process with incremental covariance $\boldsymbol{\Sigma}_1^c\,dt$, where

$$\boldsymbol{\Sigma}_1^c = \begin{pmatrix} \sigma_{11}^2 & & 0 \\ & \ddots & \\ 0 & & \sigma_{nn}^{22} \end{pmatrix}$$

and c indicate continuous time. The model expressed by Eqn (6) is a linear stochastic state-space model in continuous time.

The noise process, $d\boldsymbol{w}(t)$ in Eqn (6) accounts for deviations between the simplified model expressed by Eqn (5) and the true system. For instance, Eqn (5) does not include a latent heat component. Furthermore, the noise process accounts for disturbances in the input variables, and for disturbances of unrecognized and non-modelled input variables, e.g. wind speed and direction.

Equation (6) describes the evolution of the temperatures of all the nodes in the greenhouse, but in the present case, only the air temperature T_r is measured. Thus,

$$T_r(t) = \mathbf{CT}(t) + e(t) \tag{7}$$

where $\mathbf{C} = (1, 0, 0, \ldots)$ is a constant matrix. Hereby the air temperature is assumed to be the temperature of the first node in *Fig. 1* and Eqn (5). The measurement error $e(t)$ of the air temperature is assumed to be normally distributed white noise with zero mean and constant variance, σ_1^2. Furthermore, it is assumed that $d\boldsymbol{w}(t)$ and $e(t)$ are mutually independent.

4. Estimation

The heat dynamics in a greenhouse are modelled by Eqns (6) and (7). Let $\boldsymbol{\theta}$ denote a vector composed of the thermal parameters C_j, R_j, A_s in $\mathbf{A}$ and $\mathbf{B}$, the initial temperatures of the nodes, and the unknown variance parameters in $\boldsymbol{\Sigma}_1^c$ and σ_1^2. The parameters in $\boldsymbol{\theta}$ are estimated from data by maximizing the likelihood function

$$L(\boldsymbol{\theta}) = \prod_{t=1}^{N} \left(\frac{1}{\sigma_e(t|t-1, t-2, \ldots)\sqrt{2\pi}} \exp\left(\frac{-\varepsilon^2(t)}{2\sigma_\varepsilon^2(t|t-1, t-2, \ldots)} \right) \right) \tag{8}$$

where N is the number of observations, and t denotes the time index belonging to the set $\{1, 2, \ldots, N\}$. A Kalman filter is used to obtain recursive estimates of the predictive variance $\sigma_\varepsilon^2(t\,|\,t-1, t-2, \ldots)$ and the prediction error

$$\varepsilon(t) = T_r(t) - \hat{T}_r(t\,|\,t-1, t-2, \ldots) \tag{9}$$

The prediction error $\varepsilon(t)$ is the error of the one-step forward prediction of the air temperature from one measurement to the next. Given previous measurements, $\hat{T}_r(t\,|\,t-1, t-2, \ldots)$ denote the one-step forward prediction of the air temperature, $\hat{T}_r(t)$ in the greenhouse. For further information about the estimation procedure see Refs. 15 or 16.

5. The experiment

In order to obtain estimates of the parameters, an experiment was conducted from 1 February to 7 March 1992 at the Department of Ornamentals, Årslev, Denmark, in accordance with the experiment in Nielsen and Madsen.[13] This part of the year was chosen because a typical winter period with a typical demand on heat supply was required. The greenhouse was smaller than commercial greenhouses, being 8 m × 21·5 m in base area. It was constructed of steel with glass in aluminium frames, where the distance between the frames was 0·8 m. The greenhouse was placed in an area with other greenhouses used for experiments. During the experiment, the windows were kept closed.

In the greenhouse, there were four benches filled with pot plants (*Tagetes erecta* "*Hawaii*"). The water-based heating system consisted of two pipe heating systems, with horizontal pipes running along the length of the greenhouse. The largest heating system was placed towards the top of the house, along the side wall, and was called the wall/top heating system. The other heating system was placed near the floor, and was called the floor heating system.

Under normal conditions, the control system introduces a correlation between the solar radiation and the energy supplied by the heating system because the control system attempts to maintain a constant air temperature. This correlation introduces a difficulty at the identification stage. Hence, in order to avoid the correlation between the heat input and other input variables, the energy from the heating system was controlled by a pseudo-random binary signal (PRBS).[13] The signal determines the switch between the two levels of energy input into the greenhouse. The advantage of using a PRBS is that it has an autocorrelation close to white noise and hence is not correlated with other input signals.

The two-level strategy of heating was obtained in the following way. The flow of water was kept constant, and the inlet water temperature for the two systems, i.e. the wall/top and floor heating, was the same and controlled by the PRBS, which then determined whether the heat supply should be on the high or the low level. In order to obtain the two levels, the difference between the inlet temperature and the air temperature was kept at one level for the low heat supply and at a higher level for the high heat supply. For further information about the experiment design; see Ref. 15.

5.1. *The data*

The air temperature in the greenhouse (T_r) was measured in the middle of the house 1·5 m above the floor and behind an aspirated screen with a platinum resistance temperature sensor. The temperature in the heating system was measured with paired platinum resistance temperature sensors. A sensor was placed to measure the inlet water temperature and another sensor was placed to measure the outlet water temperature. The actual energy input of the heating system to the greenhouse (ϕ_h) is assumed to be proportional to a mean temperature difference between the actual mean temperature of the pipes in the heating systems and the actual air temperature in the greenhouse.

The solar radiation (ϕ_s) was measured with a Kipp and Zone's solarimeter placed on the top of the roof on a neighbouring greenhouse. The outdoor temperature (T_0) was measured in a Stevenson Screen.

Data from the experiment were sampled every 2 min, yielding 25 920 observations of each variable, T_0, ϕ_s, ϕ_h, T_r. Data from 3 February (720 observations) are shown in *Fig. 2.*

The plot of ϕ_h, (*Fig. 2*) shows how the PRBS determines the heat supply. On the high level the heat supply rises about 70 W/m^2 compared to the low level situation. The increase in heat supply leads to an increase in the measured air temperature, T_r in the greenhouse. Also an increase in solar radiation ϕ_s implies an increasing T_r. On 3 February, the outdoor temperature T_0 was almost constant.

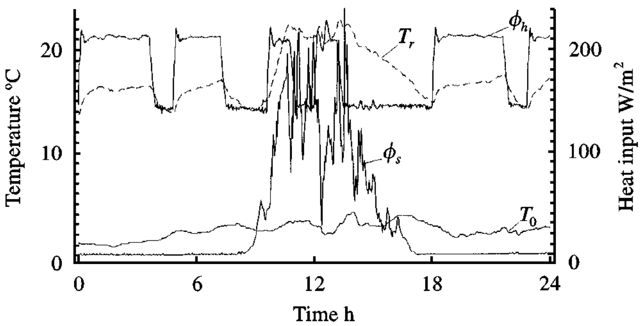

Fig. 2. Air temperature in the greenhouse (T_r), outdoor temperature (T_0), energy input of the heating system (ϕ_h) and the solar radiation (ϕ_s) recorded on 3 February

6. Results

In Table 1, some statistics of the most attractive models are shown. The first column in Table 1 contains the model number, the second column is the number of nodes and p, in the third column is the number of parameters in each model. The six parameters in model 1 are associated with Eqns (6) and (7), i.e. C_1, R_0, A_s, variance parameters σ_{11}^2 and σ_1^2 belonging to $w(t)$ and $e(t)$, plus an initial temperature of the node. Increasing the number of nodes by one increases the maximum number of parameters by four since one capacity, one resistance, one initial temperature and one error parameter of the new node are added. All the models in Table 1 relate to *Fig. 1*, where the number of nodes is the only difference. Considering model 4, the number of parameters is 16 since, nodes 3 and 4 are assumed to have the same heat capacity. Furthermore, the resistance to heat transfer between the nodes is assumed to be identical to the resistance to heat transfer between nodes 2 and 3. In model 5, all the nodes have different heat capacities and the resistances to heat transfer are different. Hence, the number of parameters in the model are 18.

Column 4 in Table 1 shows the value of the logarithm of the likelihood function L evaluated at the parameter estimates. To facilitate comparisons of the different models, the Schwartz's Bayesian criterion (SBC) is also shown. This is defined by Schwarz[17] as

$$\mathrm{SBC} = p \log N - 2 \log(L) \tag{10}$$

where N is the number of observations, p is the number of parameters and L is the likelihood value. The best model is the one which has the smallest value of SBC (note that these values are negative).

Considering the SBC values in Table 1, model 3 is better than the other models owing to the smallest SBC value. Estimates of the thermal parameters from model 3 are shown in Table 2, and the variance parameters are shown in Table 3. The table shows that T_1 is predicted with the smallest variance, whereas T_3 is predicted with the largest variance. The estimated initial temperatures at the nodes are shown in Table 4. The relatively high initial temperature of the third node shown in Table 4 describes most likely a higher than expected measured air temperature on the first day of the experiment. The correlation matrix between the parameters were also calculated, but no correlation, which could indicate an

Table 1
Statistic of the models considered

Model no.	*No. of nodes*	*No. of parameter*	*log(L)*	*SBC*
1	1	6	22 058	− 44 055
2	2	10	34 680	− 69 258
3	3	14	34 833	− 69 523
4	4	16	34 836	− 69 509
5	4	18	34 851	− 69 519

L = likelihood function; SBC = Schwartz's Bayesian Criterion.

Table 2
Estimate and standard deviation (S.D.) of thermal parameters

Parameter	*Est.*	*S.D.*
C_1, 10^6 JK^{-1}	4·79	0·04
C_2, 10^6 JK^{-1}	6·9	0·4
C_3, 10^6 JK^{-1}	46·2	1·7
R_0, 10^{-6} KW^{-1}	452	9
R_1, 10^{-6} KW^{-1}	435	12
R_2, 10^{-6} KW^{-1}	1205	53
A_s, m^2	98·8	1·2

Table 3
Estimate and standard deviation (S.D.) of variance parameters

Parameter	*Est.*	*S.D.*
σ_{11}^2, 10^{-3} K^2,	0·013	0·0005
σ_{22}^2, 10^{-3} K^2,	0·296	0·018
σ_{33}^2, 10^{-3} K^2,	1·64	0·12
σ_1^2, 10^{-3} K^2,	1·13	0·03

Table 4
Initial value of the temperature in the three nodes

	Est.	*S.D.*
T_1, °C	20·4	0·1
T_2, °C	23·6	0·9
T_3, °C	32·5	3·9

overparameterization of the model, was observed. The largest correlation coefficient is 0·8.

7. Discussion

Table 2 suggests that the model with three lumped capacitances in the three coupled differential equations is better than the other models. Hence, according to the used information criteria (SBC) a model with three thermal capacitances gives the most appropriate description of the air temperature in the greenhouse.

The model is a stochastic linear state-space model in continuous time, which takes the heat supply, solar radiation and outdoor temperature as input variables. It is expected that the wind speed has some influence on the dynamical model, and more specifically the resistance (R_0) of heat transfer from the indoor air to the outdoor air may depend on the wind speed.[5,6,9] This leads, however, to a non-linear model which makes the identification much more complicated.[16] The wind effect constitutes a subject for identification by itself.

The solar radiation is considered as an input to the first node of the models shown in Table 2. Since solar radiation does not interact directly with the greenhouse air, the input $A_s\phi_s$ in Eqn (5) can be omitted from the first differential equation and instead included in the second differential equation. But the alternative model introduces a slower heat transfer from the solar radiation, and studies have shown that the suggested model expressed by Eqn (5) has the best performance with smaller SBC values.

Based on the assumptions for $d\boldsymbol{w}(t)$, and $e(t)$ the prediction error $\varepsilon(t)$ should be a white noise. Investigations of $\varepsilon(t)$ indicate that the constant variance of $\sigma_\varepsilon^2 = 0{\cdot}062^2$, the autocorrelation function and the cross-correlation functions to the input variables show that $\varepsilon(t)$ is close to white noise without a correlation to the inputs.

An important part of the evaluation of the model is to compare the estimated parameters with parameters determined by using physical constants for the greenhouse. In the following, it is shown how the estimated parameters correspond with the physically determined values.

The loss of heat from the greenhouse to the environment is most frequently described by the heat loss transmission coefficient called the U value (W/m^{2}K). The whole area of the wall and roof of the greenhouse is 325 m^2. If the whole area of the surface is assumed to be homogeneous, then from the estimate of R_0 in Table 2 of 452 μK/W, an average U value of the greenhouse becomes 6·8 W/m^{2}K. For the greenhouse considered, this is consistent with earlier measurements of the heat consumption coefficient which varies between 0 and 9 W/m^{2}K depending of the outdoor climate.[18]

The volume of the greenhouse is 1148 m^3. Thus, at 20°C and 90% relative humidity a rough calculation of the heat capacity of the air in the greenhouse is 1·6 MJ/K. The estimate of C_1 in Table 2 is 4·79 MJ/K. Hence, the estimated value of C_1 contains more than the heat capacity of the air in the house. The boundary between the different nodes in a lumped system is not clear.[14] The estimated thermal mass of C_1 in Table 2 is the thermal mass of the greenhouse that has a uniform spatial temperature equal to the measured air temperature. The estimate of C_1 may then contain the thermal mass of the air in the greenhouse plus the thermal mass of a part of the plants, the surfaces of other objects in the house like the pots, the benches and the floor. A similar conclusion was given for ordinary buildings by Madsen and Holst.[16] The fact that C_1 estimates the thermal mass of the air and the surfaces of other objects in the greenhouse demonstrates that the heat input from the solar radiation must be added to the first differential equation in Eqn (5).

The estimated values of the heat capacities, C_2 and C_3, are complex to validate. However, let the specific heat capacity of the soil be 2 MJ/Km^3, then the soil to a depth of 1 cm has a heat capacity of 3·4 MJ/K. The estimated C_2 in Table 2 may then contain the thermal mass of the inner parts of the plants, the soil in the pots, the inner parts of the bench and a few centimetres of the ground. All the heat capacities included in C_2 are not included in C_1.

The estimated value of C_3 in Table 2 most likely corresponds to the heat capacity of a deeper part of the ground. In the table, C_3 is 46·2 MJ/K. Compared with the heat capacity of the soil, it is found that the third node in the model corresponds to a layer of about 14 cm.

The estimate of A_s in Table 2 is 98·8 m^2 and corresponds with the experimentally evaluated values of A_s, aiming at values between 81 and 210 m^2.[19]

Using the estimated parameters shown in Table 2, the temperatures in the nodes are simulated using independent data collected over a period of 2 d, 4 d after the period used for the estimation. The period was from 12:00, 11 March to 12:00, 13 March. The initial temperature of T_1 in the first node was copied from the starting point of measured air temperature T_r. For the other two nodes, likely initial temperatures were assumed. The pattern of simulated temperatures at the nodes is shown in *Fig. 3.* In general, the pattern of the simulated temperatures, T_1 in the first node shows agreement with the measured air temperature T_r in the greenhouse. The temperature of the second node T_2 shows less variation than T_1, and the variation of T_3 is smaller than the variation of T_2. The small variation of T_3 suggests that the third node most likely represents a deeper part of the ground in the house as argued previously.

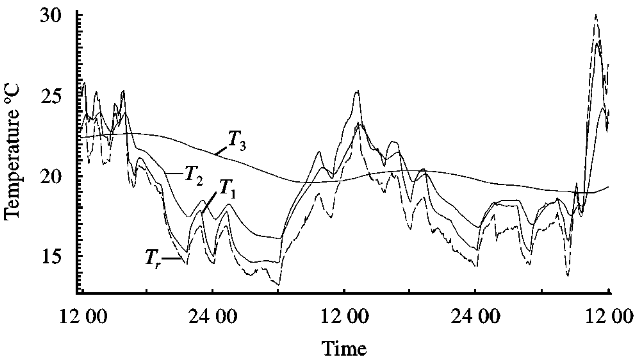

Fig. 3. Simulation of the temperatures at the three nodes (T_1, T_2, T_3) and the measured air temperature (T_r), during the two day period from 12 : 00 on 11 March to 12 : 00 on 13 March

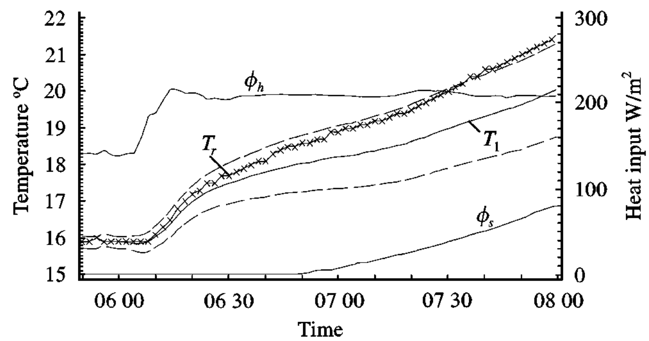

Fig. 4. Measured heat input from the heating system (ϕ_h), the solar radiation (ϕ_s), the measured air temperature (T_r), and simulated air temperatures (T_1) recorded with the 95% confidence limit shown by the dashed line

In *Fig. 3*, the pattern of the temperatures in the nodes illustrates that nodes 2 and 3 are heat accumulating thermal masses which accumulate or deliver heat, respectively, depending on the air temperature in the greenhouse. During the day when the air temperature in the greenhouse is high, the second and third nodes accumulate energy because of increasing temperatures (T_2 and T_3). However, just after sunset, at 16:30, when the air temperature decreases, the second node has a higher temperature than the air ($T_2 > T_1$) and the third node has a higher temperature than the second node ($T_3 > T_2$), hence the third node delivers heat to the second node and the second node delivers heat to the air.

One purpose of setting up a model for the heat dynamics in a greenhouse is to be able to control the heat supply in an efficient way. *Figure 4* shows how the estimated model predicts the measured air temperature, T_r, in the greenhouse on 9 March from sunrise when the heat supply from solar radiation increases and the heat demanded from the heating system decreases. These data were recorded 2 d after the period used for estimation of the model. In the figure, the heat supply, ϕ_h was kept at a constant level until 06 : 00. The heat supply was then switched to a higher constant level. Due to the sunrise, the heat supply from solar radiation, ϕ_s increased from 0 W/m^2 to 80 W/m^2 in the time from 06 : 50 to 08 : 00. At 05 : 50, some reasonable initial temperatures of T_2 and T_3 were assumed. Until 06 : 00, T_1 is updated by the measured air temperature, T_r and the model is updated by the inputs i.e. outdoor temperature, solar radiation and heat supply. After 06 : 00, only the inputs are used to predict the air temperature until 08 : 00. The increase in the air temperature is well approximated by T_1 during the first period after 06 : 00. Over time, the predicted air temperature diverges from the measured air temperature and, after 07 : 30, the measured air temperature is outside the upper 95% confidence limit. In this case, the predicted air temperature is smaller than the measured air temperature, most likely due to some autocorrelated disturbance which in turn might be due to the low wind speed, for example while, in other cases, the predicted air temperature is larger than the measured air temperature (see *Fig. 3*).

In on-line control applications, such as the generalized predictive controller, the state of each node is estimated recursively, and correspondingly the on-line estimate is as accurate as possible. The needed prediction horizon depends on the controller, the time constants and time delays of the heating system.

8. Conclusion

A linear stochastic model of the heat dynamics of a greenhouse in continuous time is identified using statistical methods. The model is useful for predicting changes in the air temperature and the stochastic part of the model provides variance and confidence levels of the predicted air temperature. The deterministic part of the model consists of three coupled differential equations.

Although a greenhouse is a complex system, it is shown that the heat dynamics can be described approximately by a few nodes. It is demonstrated that a linear model with only three nodes can describe the internal air temperature in a greenhouse with inputs only from the global radiation, the outdoor air temperature and the heat supply.

The simulated value of the temperature in the first node fits the measured air temperature of the greenhouse. The estimated heat capacity of this node corresponds to the heat capacity of the air in the greenhouse plus the thermal mass of part of the plants, the surfaces of other objects in the house, e.g. the pots, the benches and the floor. The second and the third nodes describe heat capacities which accumulate or deliver the heat, respectively, depending on the air temperature of the greenhouse.

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